

# A SUBBAND ADAPTIVE FILTERING ALGORITHM WITH ADAPTIVE INTERSUBBAND TAP ASSIGNMENT

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## 概要

新しいサブバンド適応フィルタアルゴリズムについて報告する。各サブバンド適応フィルタのタップ数は、係数絶対値または係数二乗値に基づいて制御される。全タップ数を一定に保ちつつ、冗長なタップはタップ数が不足しているサブバンドに再配分される。白色信号によるシミュレーション結果を用いて、各サブバンド適応フィルタのタップ数が各サブバンド適応フィルタの収束と共に最適値に近づくことを示す。有色信号に対しても、タップ数は白色信号の場合と同様に、安定に制御される。

## ABSTRACT

This paper presents a new subband adaptive filtering algorithm for adaptive FIR filters. The number of taps for each subband filter is adaptively controlled based on a sum of the absolute coefficients or the coefficient power. Keeping the total number of taps constant, redundant taps are redistributed to subbands where the number of taps is insufficient. Simulation results with a white signal show that the number of taps in each subband approaches an optimum as each subband filter converges. For a colored signal, tap assignment by the new algorithm is as stable as for a white signal.

## 1. INTRODUCTION

Adaptive FIR Filters are commonly used for a wide variety of applications because of

its guaranteed stability and well-investigated behavior. However, in some applications such as acoustic echo cancelers, a large number of taps are required to cancel an echo cause by the long tail of the room impulse response. Assuming 8 kHz sampling and a five-hundred-millisecond impulse-response length, which is commonly encountered in a teleconferencing room, the number of necessary taps amounts to four thousand [1]. More over, when the sampling rate is increased to cover a wider bandwidth for better audio/speech quality, the number of taps becomes more serious problem.

To overcome this obstacle, subband adaptive filters have been applied [2]-[5]. The input signal to the adaptive filter is split into several subbands by a filterbank and then processed by independent adaptive filters in each subband. The input signal is subsampled after subdivision, and thus, required computation is reduced. The decimation rate may be selected as  $M/K$ , where  $M$  and  $K$  are positive integers greater than zero. When  $M$  is equal to the number of subbands and  $K=1$ , input signals to subband adaptive filters are critically sampled [3]. For alias-free subband adaptive filtering, oversampled structure is proposed where  $K>1$  [4],[5]. For an  $M$ -band subband adaptive filter, the necessary amount of processing is reduced by  $M/K$ .

Most of the reported subband adaptive filters are equipped with an adaptive filter with

equal number of taps for each subband. However, an actual room impulse response has a shorter length in a higher subband [6]. This fact suggests that required total computation for all subbands may be further reduced by assigning different number of taps for each subband adaptive filter. Based on this observation, a subband adaptive filter with automatic tap assignment to each subband has been proposed [7]. As the number of taps is controlled in terms of the subband residual-echo power, tap-number control may not be correct for a colored signal where powers in different subbands are not identical.

The purpose of this paper is to present another algorithm for adaptive intersubband tap assignment. The number of taps in each subband is controlled based on the coefficient values which approximate the tail of the room impulse response. In the following section, a room impulse response is analysed, followed by a review of subband adaptive filtering based on [7]. A new tap assignment algorithm is then introduced. Finally, performance of [7] and the new algorithm are compared by simulation results for acoustic echo cancellation.

## 2. ANALYSIS OF A ROOM IMPULSE RESPONSE

A room impulse response of a medium-size conference room sampled at 32 kHz is depicted in Fig. 1 with its power spectrum. The bandwidth is 16 kHz. The power spectrum is small in magnitude between 0.38 and 0.5 in the normalized frequency. The power of the impulse response corresponding to this range is naturally supposed to be small, hence, necessitates a smaller number of taps.

Frequency decomposition of the impulse response is shown in Fig. 2 for a four-band case. The bandwidth of each subband is 4 kHz. It is clearly observed that a smaller number of coefficients are necessary for a higher subband. As was seen in the power spectrum in Fig. 1, the fourth subband which corresponds to 0.375-0.5 band in Fig. 1 needs much shorter length for the adaptive filter. Rough estimates of the required number of taps are compared for the uniform and the nonuniform coefficient

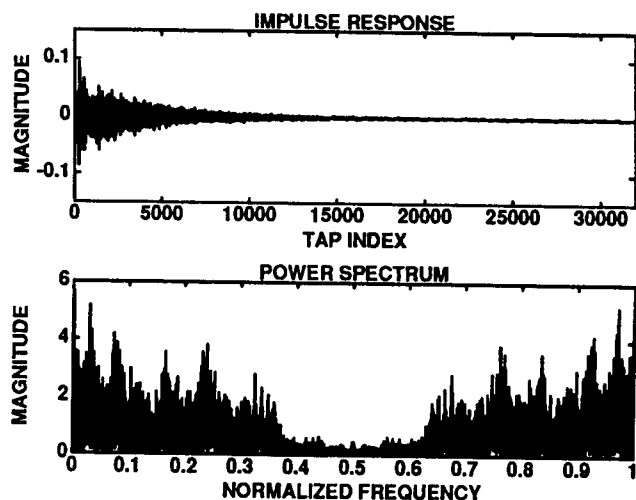


Fig. 1. A Room Impulse Response and Its Power Spectrum.

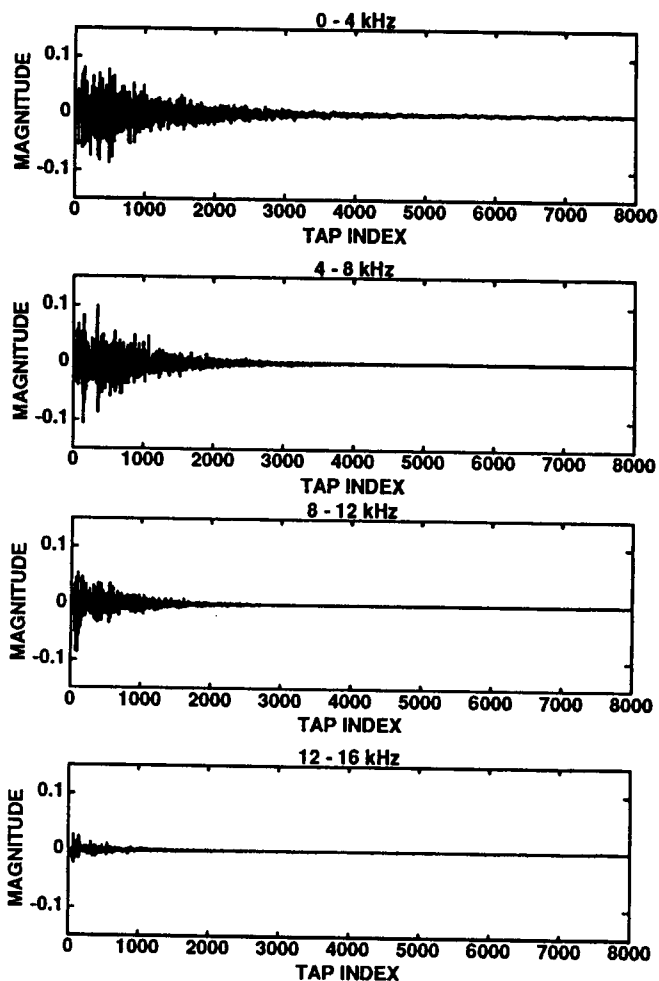


Fig. 2. Frequency Decomposition of the Impulse Response.

distribution in Tab. 1. A rough estimate in Tab.

Tab. 1 Comparison of Required Number of Taps.

DISTRIBUTION	SUBBAND	TAPS
Uniform	1	8000
	2	8000
	3	8000
	4	8000
	TOTAL	32000
Nonuniform	1	8000
	2	5000
	3	4000
	4	2000
	TOTAL	19000

1 suggests that 40% of the coefficients could be saved if intersubband tap assignment were perfect.

### 3. TAP ASSIGNMENT BASED ON THE MISADJUSTMENT

#### 3.1 Algorithm

Typical system identification by subband adaptive filtering is illustrated in Fig. 3 for an M-band case. The input signal to the unknown system is divided into subbands by an analysis filter bank to serve as the reference signal for each subband adaptive filter. The output signal from the unknown system is also divided and used as the desired signal in subbands. The misadjustments or errors in each subband are combined by a synthesis filter bank to form a full-band misadjustment. When identification is carried out perfectly, the full-band misadjustment should be zero. Tap assignment is controlled in TAP ASSIGNMENT CONTROL block based on some criterion.

Let us define some important signals. The coefficient vector  $c_{i,k}$  and the reference signal vector  $x_{i,k}$  in the  $i$ -th subband ( $i=1,2,\dots,M$ ) are determined by

$$c_{i,k} = [c_{i,1,k} \ c_{i,2,k} \ \dots \ c_{i,N_{i,k}}]^T, \quad (1)$$

$$x_{i,k} = [x_{i,k-N_{i,k}+1} \ x_{i,k-N_{i,k}+2} \ \dots \ x_{i,k}]^T, \quad (2)$$

where  $[\cdot]^T$  denotes vector transpose.  $N_{i,k}$

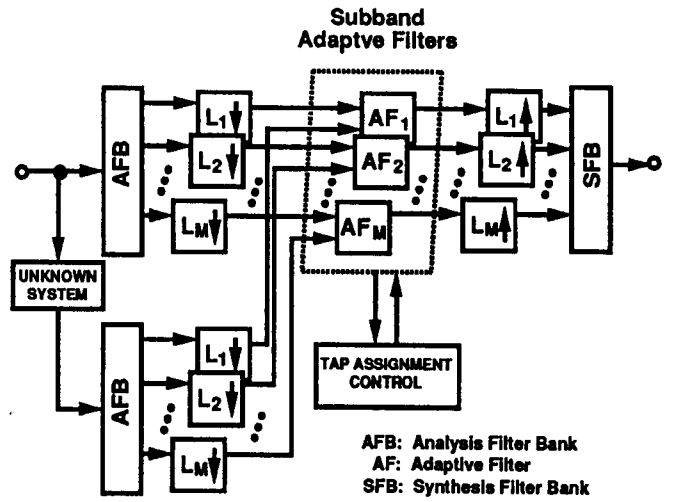


Fig. 3. Subband Adaptive Filtering.

stands for the number of taps in the  $i$ -th subband at the  $k$ -th coefficient adaptation. The subband misadjustment  $e_{i,k}$  is then given by

$$e_{i,k} = y_{i,k} - c_{i,k}^T x_{i,k}, \quad (3)$$

where  $y_{i,k}$  is the desired signal in the  $i$ -th subband, or in other words, the unknown system output decomposed into the  $i$ -th subband.

Tap assignment for a two-band case presented in [7] adjusts the number of taps for each subband by the following equations.

$$\begin{cases} N_{1,k+1} = N_{1,k} + \text{sign}(\bar{e}_{1,k} - \bar{e}_{2,k}) \\ N_{2,k+1} = N_{2,k} + \text{sign}(\bar{e}_{2,k} - \bar{e}_{1,k}) \end{cases} \quad (4)$$

$\text{sign}(\cdot)$  is equal to the sign of a nonzero argument and zero for a zero argument. A segmental sum of the squared misadjustment  $\bar{e}_{i,k}$  ( $K=1,2$ ) is defined by

$$\bar{e}_{i,k} = \sum_{j=k-P+1}^k e_{i,j}^2, \quad (5)$$

where  $P$  is a positive integer. M-band extensions of the algorithm by both the squared misadjustment and the absolute misadjustment are disclosed in [8].

Coefficients in each subband are updated by normalized LMS (NLMS) algorithm [9]. It is given by

$$c_{i,k+1} = c_{i,k} + \frac{\mu}{\delta + x_{i,k}^T x_{i,k}} e_{i,k} x_{i,k}. \quad (6)$$

$\mu$  is the step size for coefficient adaptation and  $\delta$  is a small positive number to avoid division by zero when the reference signal samples are zero.

(4) means that the adaptive filter in the first subband will have an additional tap if the misadjustment is larger than in the second subband. On the contrary, the number of taps is increased if the misadjustment is smaller than in the second subband. The same control of the number of taps is carried out in the second subband. Modification of the number of taps is performed single tap at a time.

### 3.2. Optimum Number of Taps

Assuming that the equivalent impulse response in the  $i$ -th subband is  $g_i$ , (3) becomes (7).

$$e_{i,k} = (g_i - c_{i,k})^T x_{i,k} \quad (7)$$

The mathematical expectation of the squared  $i$ -th subband misadjustment  $E[e_{i,k}^2]$  is then given by

$$E[e_{i,k}^2] = E[(g_i - c_{i,k})^T x_{i,k} x_{i,k}^T (g_i - c_{i,k})]. \quad (8)$$

It is commonly assumed for analysis that  $x_{i,k}$  and  $c_{i,k}$  are statistically independent. In addition, for simplicity,  $x_{i,k}$  is assumed to be zero-mean and white. With these assumptions, (8) reduces to

$$E[e_{i,k}^2] = \sigma_{i,k}^2 E[(g_i - c_{i,k})^T (g_i - c_{i,k})], \quad (9)$$

where  $\sigma_{i,k}^2$  is the variance of  $x_{i,k}$ .

$$\sigma_{i,k}^2 = E[x_{i,k}^T x_{i,k}]. \quad (10)$$

Let us assume that in the  $i$ -th subband, the number of taps is insufficient. Under this assumption,  $c_{i,k}$  could be redefined as

$$c_{i,k} = [h_{i,k}^T \quad 0^T]^T. \quad (11)$$

$0$  is a column vector whose dimension is the difference between the dimensions of  $c_{i,k}$  and  $h_{i,k}$ .  $g_i$  could be divided into two matrices that have the same dimensions as  $h_{i,k}$  and  $0$ .

$$g_i = [g_{1,i}^T \quad g_{2,i}^T]^T \quad (12)$$

Substituting (11) and (12) into (9) yields

$$E[e_{i,k}^2] = \sigma_{i,k}^2 E[(g_{1,i} - h_{i,k})^T (g_{1,i} - h_{i,k})]$$

$$+ \sigma_{i,k}^2 E[g_{2,i}^T g_{2,i}] = \sigma_{i,k}^2 (\alpha_{i,k} + \beta_{i,k}). \quad (13)$$

$\alpha_{i,k}$  and  $\beta_{i,k}$  are defined by

$$\begin{cases} \alpha_{i,k} = E[(g_{1,i} - h_{i,k})^T (g_{1,i} - h_{i,k})] \\ \beta_{i,k} = E[g_{2,i}^T g_{2,i}] \end{cases} \quad (14)$$

They represent misadjustment based on immature coefficients and misadjustment based on tap shortage, respectively. After convergence,  $h_{i,k}$  should be equal to  $g_{1,i}$  making  $\alpha_{i,k}$  minimum. In the ideal case, it would be equal to zero. Therefore, (13) reduces to

$$E[e_{i,k}^2] \approx \sigma_{i,k}^2 \cdot \beta_{i,k}. \quad (15)$$

It is seen from (15) that the misadjustment in the  $i$ -th subband depends on the subband-signal power  $\sigma_{i,k}^2$  and the misadjustment caused by tap shortage. Therefore, the optimum number of taps for subbands are obtained by equating the right-hand side of (15) for all subbands, if the impulse response of the unknown system, the total number of taps and all the subband-signal powers are known *a priori*.

For a white signal, of course  $\sigma_{i,k}^2$ 's are unnecessary as they are identical. Thus, finding the optimum number of taps is reduced to finding the number of taps  $N_{s,i}$  from the tail of the response which satisfies (16) for  $i = 1, 2, \dots, M$ .

$$\sum_{j=KN/M-N_{s,i}+1}^{KN/M} g_{2,i,j}^2 = G. \quad (16)$$

$G$  is a positive constant.  $N$  is the number of the taps for the full-band impulse response and

$$g_{2,i,j} = [g_{2,i,1} \quad g_{2,i,2} \quad \dots \quad g_{2,i,N_{s,i}}]. \quad (17)$$

### 3.3. Factors for Incorrect Tap Assignment

In the beginning of the convergence process,  $E[e_{i,k}^2]$  is dominated by misadjustment based on immature coefficients ( $\alpha_{i,k}$ ) rather than misadjustment based on tap shortage ( $\beta_{i,k}$ ). Essential information on tap shortage or excess is not properly reflected in the early stage of convergence. This will cause incorrect control. In addition, when there is imbalance in power among subband reference signals, the

power difference enlarges the gap between the desired control and the actual control. It is apparent from (15) that tap assignment control results are different for a white and a colored signal. Therefore, intersubband tap assignment based on subband misadjustment tends to be inaccurate especially in early convergence stage and for a colored signal.

#### 4. TAP ASSIGNMENT BASED ON COEFFICIENT VALUES

In the tap assignment based on the misadjustment, the cost function for assignment is a function of subband reference signals and all the subband coefficients as in (5) and (9). To be free from the reference-signal imbalance, the reference signal is eliminated from the cost function. Ignoring the coefficients which correspond to the top of the impulse response, influence of immature coefficients is reduced. These modifications naturally lead to a tap assignment method where only the coefficients corresponding to the tail of the impulse response are evaluated.

A new cost function  $\bar{c}_k$  is introduced for the above purpose such that

$$\bar{c}_k = \sum_{j=k-P+1}^k [\bar{c}_{1,j}^T \bar{c}_{1,j} \bar{c}_{2,j}^T \bar{c}_{2,j} \cdots \bar{c}_{M,j}^T \bar{c}_{M,j}]^T. \quad (18)$$

$\bar{c}_{i,j}$  is given by

$$\bar{c}_{i,j} = [c_{i,N_{i,k}-R+1,j} \ c_{i,N_{i,k}-R+2,j} \ \cdots \ c_{i,N_{i,k},j}]^T \quad (19)$$

where R is a positive integer. The number of taps  $N_{i,k+1}$  in the i-th subband at the (k+1)-th iteration is determined by (20).

$$N_{i,k+1} = N_{i,k} - R + \Delta N_{i,k} \quad (20)$$

The increment/decrement of the number of taps for the i-th subband is

$$\Delta N_{i,k} = \text{INT} \left[ \frac{MR \times \bar{c}_{i,k}^T \bar{c}_{i,k}}{\text{trace}\{\bar{c}_k\}} \right]. \quad (21)$$

INT [·] takes an integer which is greater than and closest to the argument. As a result of this tap redistribution,

$$\sum_{i=1}^M \Delta N_{i,k} > MR \quad (22)$$

might be encountered, whereas

$$\sum_{i=1}^M \Delta N_{i,k} < MR \quad (23)$$

never happens. If

$$\sum_{i=1}^M \Delta N_{i,k} - MR = Q, \quad (24)$$

the number of taps is decreased by one in Q subbands which correspond to Q smallest values of  $\Delta N_{i,k}$ .

Adaptive intersubband tap assignment based on coefficient powers is reduced to a simpler form for a two-band case with R=1. The number of taps in the first and the second subbands are adjusted by the following equations.

$$\begin{cases} N_{1,k+1} = N_{1,k} + R \cdot \text{sign}(\Delta \bar{c}_k) \\ N_{2,k+1} = N_{2,k} - R \cdot \text{sign}(\Delta \bar{c}_k) \end{cases} \quad (25)$$

$$\Delta \bar{c}_k = \sum_{j=k-P+1}^k (\bar{c}_{1,k}^T \bar{c}_{1,k} - \bar{c}_{2,k}^T \bar{c}_{2,k}) \quad (26)$$

If the sum of the squared coefficients in the tail is larger, the number of taps is increased by R. Otherwise, it is reduced by R. (18)-(26) could easily be extended for tap assignment by absolute values of coefficient instead of coefficient powers [8].

#### 5. SIMULATION RESULTS

Simulations have been carried out with both a white *Gaussian* signal and a colored signal for the input signal to the unknown system. The white signal has zero mean and unit variance. The colored signal was generated by filtering a white *Gaussian* signal by

$$A(Z) = \frac{0.25}{1 - 1.5Z^{-1} + Z^{-2} - 0.25Z^{-3}}. \quad (27)$$

Another white *Gaussian* signal is added to the output of the unknown system as an additive noise. The additive-noise level is -40 dB in power from the unknown-system input. Over-sampled subband adaptive filtering [4],[5] is

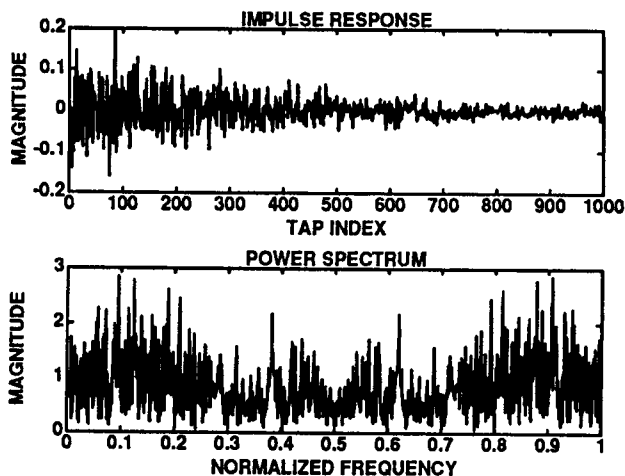


Fig. 4 Impulse Response of the Unknown System.

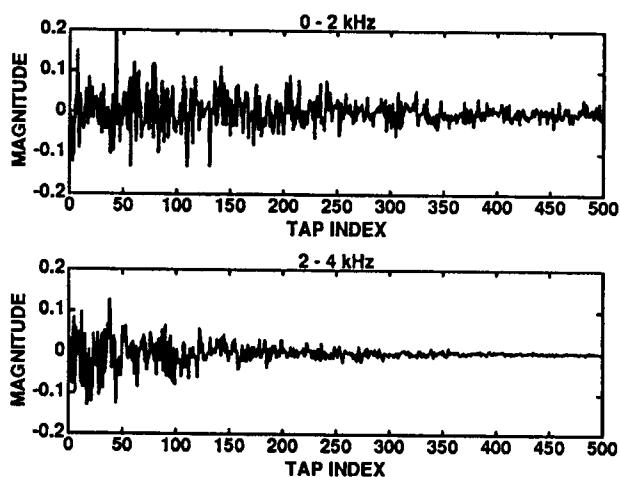


Fig. 5 Frequency Decomposition into Two Bands.

assumed unless otherwise stated. ERLE (echo return-loss enhancement) in the  $i$ -th subband is calculated by

$$ERLE_{i,k} = \frac{\sum_{j=k-N_a+1}^k \{g_i^T x_{i,k}\}^2}{\sum_{j=k-N_a+1}^k e_{i,k}^2}, \quad (28)$$

where  $N_a$  is an average number of taps given by

$$N_a = \frac{1}{M} \sum_{i=1}^M N_{i,k}. \quad (29)$$

The impulse response of the unknown system used for simulations and its power spectrum are demonstrated in Fig. 4. The number of taps

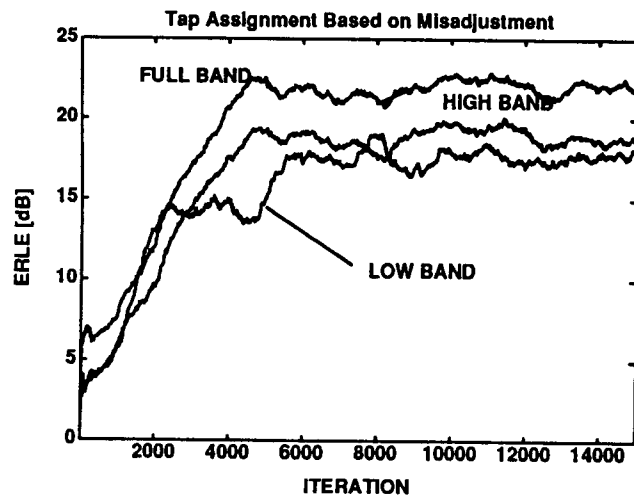


Fig. 6. Convergence of ERLE with Tap Assignment by Misadjustment for a White Signal.

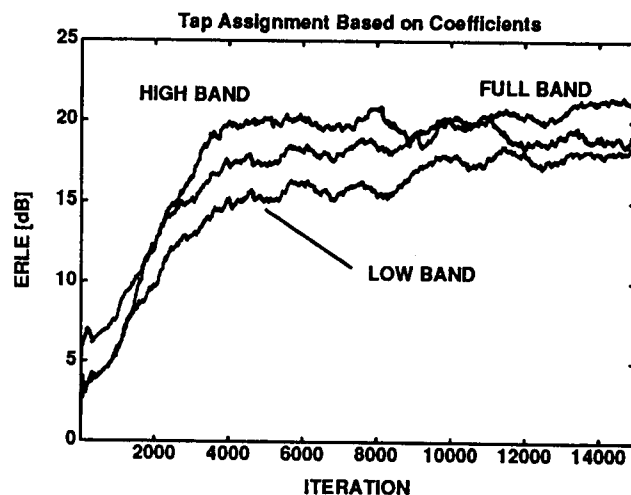


Fig. 7. Convergence of ERLE with Tap Assignment by Coefficients for a White Signal.

were selected as 1000 and 8 kHz sampling was assumed.

An artificial impulse response has been generated as follows. A white signal with an exponential envelope has been generated and split into two bands. The high-band response has further been reshaped with another exponential function to have a shorter length than the low bands. The low-band response has been kept unaltered. These two-band responses have been synthesized to recover the full-band impulse response. The low- and high-band components are depicted in Fig. 5.

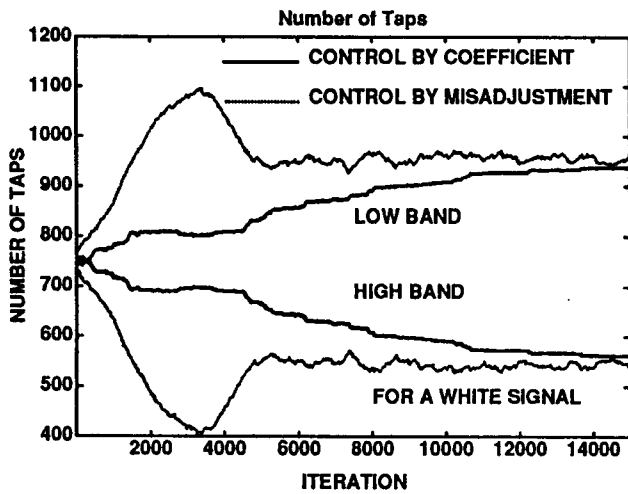


Fig. 8. Number of Taps for a White Signal.

Figure 6 exhibits convergence characteristics for a white signal in terms of ERLE by tap assignment based on the misadjustment. Figure 7 is the corresponding ERLE by tap assignment based on the coefficients. Both finally achieve approximately 22 dB. The ERLE's in the low- and the high-band are almost equal in both algorithms. This fact suggests that for a white signal input, the number of taps are controlled such that the misadjustment becomes identical in subbands. Good agreement on the converged ERLE values in Figs. 6 and 7 shows validity of (15). For a white signal input, making  $\beta_{i,k}$  same in all subbands is equivalent to having the same misadjustments. ERLE development is faster with tap assignment by the misadjustment. This is due to fast convergence of tap assignment as in Fig. 8.

Figure 8 compares tap assignment behavior by [7] and the proposed algorithm. [7] provides faster convergence for the number of taps. However, it should be noted that it is caused by an overshoot on its trajectories. The number of taps is "over-controlled" by [7] as was described in section 2.3. There is a dilemma that fast convergence is obtained by this incorrect control, which often brings undesirable results as is described in the following paragraph.

In Fig. 9, a colored signal is applied to [7]. In the beginning of the convergence, the full-band ERLE is degraded because of the

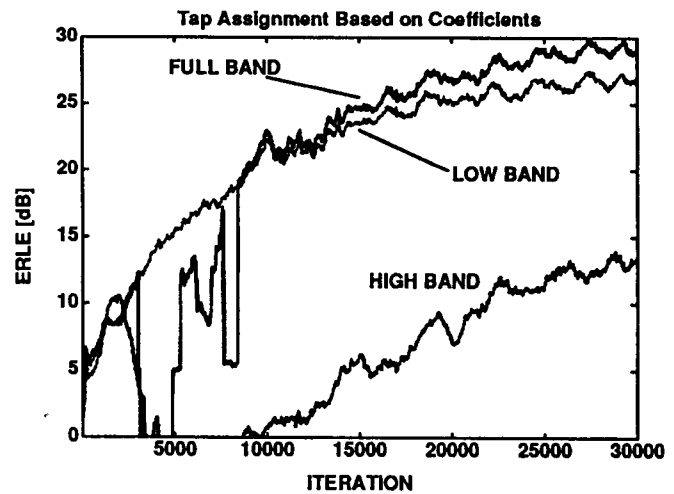


Fig. 9. Convergence of ERLE with Tap Assignment by Misadjustment for a Colored Signal.

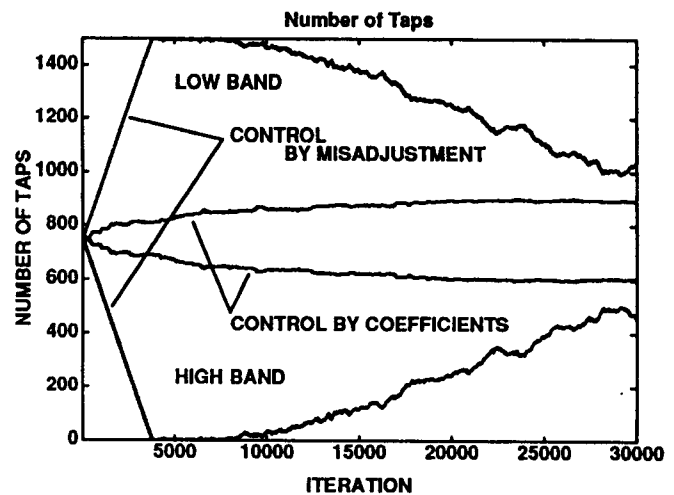


Fig. 10. Number of Taps for a Colored Signal.

undesirable overshoot described earlier. It is confirmed by tap-assignment behavior in Fig. 10. Because no tap is assigned to the high band between 4000 and 8000 in iteration, ERLE degradation in the high band gives undesirable influence to the full-band ERLE. On the other hand, the new algorithm continues to improve the ERLE with no degradation in its process as shown in Fig. 11. The number of taps is controlled smoothly as in Fig. 10. This becomes possible thanks to the tap assignment behavior with no subband-power influence.

To avoid undesirable overshoot in tap assignment by [7], upper- and lower-limit for each subband may be determined. However,

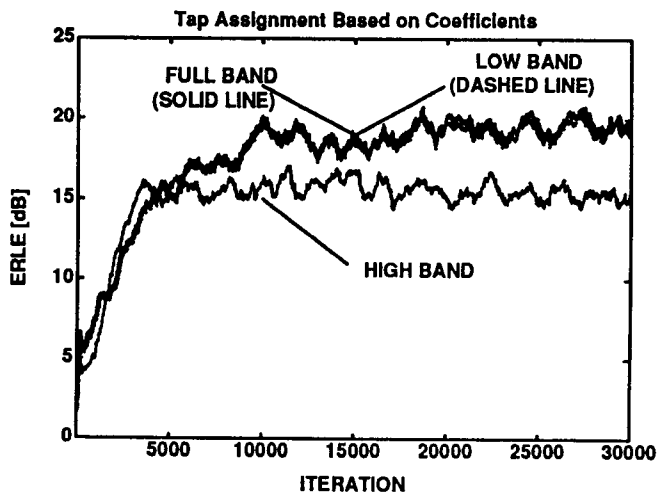


Fig. 11. Convergence of ERLE with Tap Assignment by Coefficients for a Colored Signal.

since impulse responses in each subbands are generally unknown during the convergence process, selection of the limits is not a feasible approach. In terms of feasibility, the new algorithm is superior to that by [7], although further optimization and improvements are expected. For both algorithms, it is an essential characteristic that correct tap shortage/excess could not be calculated before convergence. This point leaves a large room for further study.

## 6. CONCLUSION

A new subband adaptive filtering algorithm for adaptive FIR filters has been presented. The number of taps for each subband filter is adaptively controlled based on the coefficients instead of the misadjustment. Simulation results with a colored signal revealed that tap assignment based on the misadjustment has possibilities of "over/under assignment" by power imbalance in subbands. This has been confirmed by analytical results on the optimum tap distribution. The new algorithm does not have this problem and provides stable convergence. However, the presented results are preliminary and detailed investigations are left for further study.

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