A Modified Normalized LMS Algorithm Based on a Long-Term Average of the Reference Signal Power

参照入力信号パワーの長時間平均値に基づく正規化LMS アルゴリズム

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Abstract

This paper proposes a modified normalized LMS algorithm based on a long-term average of the reference input signal power. The reference input signal power for normalization is estimated by using two leaky integrators with a short and a long time constants. Computer simulation results compare the performance of the proposed algorithm with some previously proposed adaptive-step algorithms. The proposed algorithm converges faster than the conventional adaptive-step algorithms. Almost 30dB of the ERLE, which is comparable to the conventional algorithms, is achieved in noisy environments.

あらまし

参照入力信号の長時間平均パワーに基づく正規化 LMS アルゴリズムを提案する。正規化に用いるパワーは、時定数の異なる2種類のリーク積分を用いて推定する。実音声および自動車内で収録した雑音を用いた計算機シミュレーションによって、提案法と従来の可変ステップサイズアルゴリズムを比較する。本アルゴリズムは、雑音が混入した環境下でも、可変ステップアルゴリズムと同等の最大約 30dB というエコー抑圧量が得られている。また、本アルゴリズムは従来の可変ステップサイズアルゴリズムより速く収束する。

1. Introduction

Hands-free telephones, which make conversation without a hand-set possible, have become more popular because of their convenience. Recently, some mobile communication systems have introduced hands-free function for safety as well as convenience. In a conversation without a hand-set, acoustic echoes generated by speech propagation from a loudspeaker to a microphone disturb comfortable conversations. Echo cancellers are widely used to reduce such echoes.

Adaptive filters based on stochastic gradient algorithms[1-7] are good candidates for echo cancellers because of their simplicity. The LMS (least mean square) algorithm[1] and the learning identification method (known as the normalized LMS algorithm, NLMS) [2, 3] are the most popular examples. The LMS algorithm is not suitable for non-stationary signals because its convergence characteristics heavily depend on the reference input signal power. Thanks to the normalization by the reference input signal power, the convergence speed of the NLMS algorithm is independent of the reference signal power.

The normalization in the NLMS algorithm, however, makes the NLMS algorithm not applicable to noisy environments such as mobile hands-free telephones. The influence of the additive noise becomes too large if the reference input signal power is small[4]. On the other hand, the LMS algorithm, which can be regarded as the NLMS algorithm normalized by an infinite-term average power, is robust against noise. These facts suggest that the performance of the NLMS algorithm depends on the power estimation procedure. However, the influence of the power for normalization on the convergence characteristics has not been clarified.

This paper proposes a modified normalized LMS algorithm based on a long-term average of the reference input signal power. The signal power for normalization is estimated by using two leaky integrators with a short and a long time constants. In Section 2, the LMS algorithm, the NLMS algorithm and their problems are described. Section 3 examines the influence of the reference input power estimation on the convergence characteristics of the NLMS algorithms and show that the NLMS algorithm becomes robust against noise or even unstable depends on a power estimation parameter. The proposed algorithm with a modified power estimator is described in Section 4. The performance of the proposed algorithm are compared with three previously proposed adaptive-step algorithms [5-7] in Section 5.

2. LMS algorithm and NLMS algorithm

Assuming an N-tap FIR adaptive filter, the filter output $\hat{y}(t)$ at time t is calculated by

$$\hat{y}(t) = \mathbf{W}^T(t)\mathbf{X}(t) \tag{1}$$

where $\mathbf{W}(t)$ is the filter coefficient vector, $\mathbf{X}(t)$ is the reference input signal vector and a superscript T denotes transpose of a vector. $\mathbf{X}(t)$ contains the latest N samples of the reference input signal $x(t)$ and is given by

$$\mathbf{X}(t) = [x(t)x(t-1)\cdots x(t-N+1)]^T. \tag{2}$$

$\mathbf{W}(t)$ contains N filter coefficients and is so updated as to minimize the error signal

$$e(t) = y(t) - \hat{y}(t) \tag{3}$$

where $y(t)$ is the desired signal.

In the LMS algorithm, $\mathbf{W}(t)$ is adapted by

$$\mathbf{W}(t+1) = \mathbf{W}(t) + \mu_{LMS}e(t)\mathbf{X}(t). \tag{4}$$

A constant μ_{LMS} is so-called step-size which controls the convergence characteristics. For stationary reference input signals, the filter coefficients will converge if

$$0 < \mu_{LMS} < \frac{2}{N\sigma_X^2} \tag{5}$$

where σ_X^2 is the variance of the reference input signal $x(t)$.

The NLMS algorithm uses a time-varying parameter rather than a fixed parameter μ_{LMS}. The filter coefficients are updated by

$$\mathbf{W}(t+1) = \mathbf{W}(t) + \frac{\mu_{NLMS}e(t)\mathbf{X}(t)}{NP_X(t)} \tag{6}$$

where μ_{NLMS} is a positive constant and $P_X(t)$ is the reference input signal power. The range of μ_{NLMS} which ensure convergence is independent of $x(t)$ and is given by

$$0 < \mu_{NLMS} < 2. \tag{7}$$

The reference input signal power $P_X(t)$ can be calculated either by a squared norm[2]

$$P_X(t) = \frac{1}{N}\mathbf{X}^T(t)\mathbf{X}(t) \tag{8}$$

or by a leaky integration[3]

$$P_X(t) = aP_X(t-1) + (1-a)x^2(t). \tag{9}$$

A positive constant a controls the time constant of leaky integration. The LMS algorithm can be regarded as a special case of (9) where the time constant is infinite. The step-size μ_{LMS} becomes

$$\mu_{LMS} = \frac{\mu_{NLMS}}{NP_X(t)}, \tag{10}$$

which is derived by comparison of (4) and (6).

Both the LMS algorithm and the NLMS algorithm have their own problems. The LMS algorithm is not applicable if the reference input signal power is unknown because selection of μ_{LMS} requires a knowledge of the signal power. A most serious problem of the NLMS algorithm is the influence of the additive noise. Using the optimum filter coefficient vector $\mathbf{H}$

and the additive noise $n(t)$, adaptation of $\mathbf{W}(t)$ is given by

$$\mathbf{W}(t+1) = \mathbf{W}(t) + \frac{\mu_{NLMS}(\mathbf{H}-\mathbf{W}(t))^T\mathbf{X}(t)\mathbf{X}(t)}{P_X(t)} + \frac{\mu_{NLMS}n(t)\mathbf{X}(t)}{P_X(t)}. \quad (11)$$

Normalization by $P_X(t)$ makes the contribution of the coefficient error $\mathbf{H}-\mathbf{W}(t)$ to adaptation independent of $P_X(t)$. However, the influence of the noise $n(t)$ becomes too large for small $P_X(t)$. This is why the NLMS algorithm cannot update filter coefficients correctly if the reference input signal is non-stationary and the additive noise exists[4]. Similarly, adaptation for the LMS algorithm becomes

$$\mathbf{W}(t+1) = \mathbf{W}(t) + \mu_{LMS}(\mathbf{H}-\mathbf{W}(t))^T\mathbf{X}(t)\mathbf{X}(t) + \mu_{LMS}n(t)\mathbf{X}(t). \quad (12)$$

Since μ_{LMS} is selected so small as to satisfy (5) for largest σ_X^2, the influence of the additive noise $n(t)$ on the LMS algorithm is much smaller than that on the NLMS algorithm.

Using a leaky integration, the behavior of the NLMS algorithm can be controlled by the constant a. A small a makes the behavior similar to that using a squared norm. A behavior similar to the LMS algorithm can be obtained if a approaches 1.0. However, the influence of the power estimation on the convergence characteristics of the NLMS algorithm has not been clarified.

3. Influence of Power Estimation in NLMS Algorithm

Convergence characteristics of the NLMS algorithm has been examined for several power estimation conditions. The NLMS algorithm based on squared norm (NLMS/Norm), the NLMS algorithm based on leaky integration (NLMS/Leak) and the LMS algorithm have been compared by computer simulations for acoustic echo cancellation in mobile hands-free telephones. The echo and the additive noise have been recorded in a car. The reference input signal and the noise are a female speech and an idling noise by a diesel engine. The maximum echo-to-noise ratio (ENR) is about 20dB. The number of taps is 512. The step sizes μ_{LMS} and μ_{NLMS} have been optimized in order to achieve largest ERLE (Echo Return Loss Enhancement). μ_{LMS} and μ_{NLMS} are chosen as 1.25×10^{-10} and 0.1, respectively.

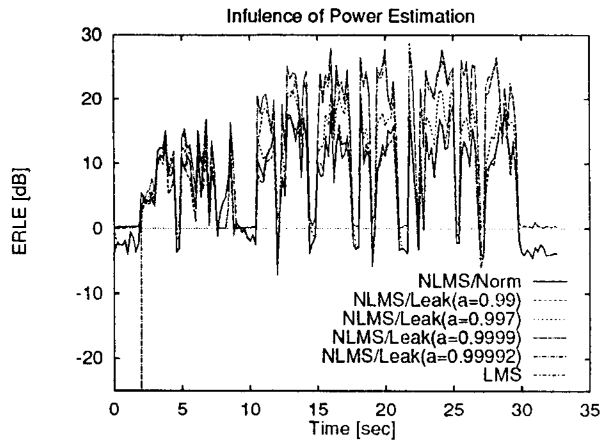

Fig. 1. Influence of power for normalization.

Figure 1 compares the ERLE of the LMS algorithm and the NLMS algorithm for several power estimation parameters. The performance of the LMS algorithm is superior than NLMS/Norm. The ERLE of NLMS/Leak depends on the constant a; better ERLE is achieved with larger a. The convergence characteristics of NLMS/Leak with a large a is similar to the LMS algorithm, which can be considered as NLMS/Leak with infinite time constant. Though larger a provides better performance, too large a makes NLMS/Leak unstable. Figure 1 shows that NLMS/Leak is unstable if $0.99992 \le a$; the ERLE for $a = 0.99992$ becomes $-\infty$ around 2 seconds.

The estimated powers $NP_X(t)$ are shown in Fig. 2. The power for the LMS algorithm

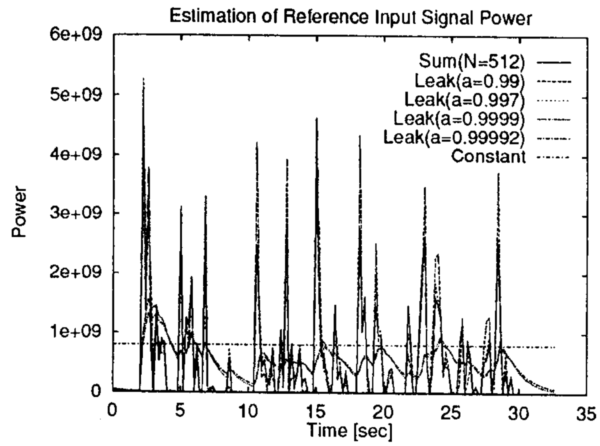

Fig. 2. Power for normalization.

can be considered as a constant and derived from (10) as

$$NP_X(t) = \frac{\mu_{NLMS}}{\mu_{LMS}}. \tag{13}$$

For large a, the tracking speed of the leaky integration becomes slower and the estimated power keeps larger value even in no-speech periods. This prevents normalization by too small power and enhances robustness against the additive noise. On the other hand, too small estimated power in large-power periods makes LMS/Leak unstable. However, the condition to select a has not been clarified.

4. NLMS Algorithm Based on Long-Term Average Power

For robustness against the additive noise, NLMS/Leak with a larger time constant is a good candidate if convergence is guaranteed. In order to ensure convergence of NLMS/Leak, the reference input signal power $P_X(t)$ should satisfy

$$\frac{\mu_{NLMS}}{NP_X(t)} < \frac{2}{N\sigma_X^2}. \tag{14}$$

The lower limit on $P_X(t)$ is

$$P_{XTh}(t) = \frac{\mu_{NLMS}\sigma_X^2}{2} \tag{15}$$

which can be smaller than σ_X^2. If $P_X(t)$ does not satisfy (15), $P_X(t)$ should be replaced by an appropriate value, e.g. a lower threshold $P_{XTh}(t)$. For non-stationary signals, σ_X^2 should be estimated by another power estimator with fast tracking capability.

Proposed algorithm uses two power estimators with different tracking speed for both robustness against the noise and stability. The short term average power $P_{XS}(t)$ is calculated by

$$P_{XS}(t+1) = a_s P_{XS}(t) + (1-a_s)x^2(t). \tag{16}$$

A conditional leaky integrator with a long time constant calculates the reference input signal power $P_{XL}(t)$. To avoid convergence of $P_{XL}(t)$ to zero, $P_{XL}(t)$ is updated only if $c_1 P_{XL}(t) \le P_{XS}(t)$ by

$$P_{XL}(t+1) = a_L P_{XL}(t) + (1-a_L)x^2(t). \tag{17}$$

Constants a_L and a_s are chosen to satisfy $0 < a_s < a_L < 1$. A constant c_1 for the threshold should be $0 < c_1 < 1$. Then, $P_{XL}(t+1)$ is compared with the lower threshold $Th_L(t)$. If $P_{XL}(t+1) < Th_L(t)$, $P_{XL}(t+1)$ is replaced by $Th_L(t)$. The threshold $Th_L(t)$ is calculated by

$$Th_L(t) = c_2 P_{XS}(t). \tag{18}$$

where c_2 is a positive constant which satisfies

$$\frac{\mu_{NLMS}}{2} < c_2. \tag{19}$$

Using $P_{XL}(t)$ calculated by this procedure, the filter coefficients $\mathbf{W}(t)$ are updated in the same manner as the NLMS algorithm. Note that the proposed algorithm requires only a small number of additional computation to the NLMS algorithm. In some applications such as acoustic echo cancellers, a large number of taps over several hundreds makes the total amount of computation for the proposed algorithm almost same as the NLMS algorithm.

5. Computer Simulations

Computer simulations have been carried out for the same manner as in Section 3. Another noise recorded in a moving car, which

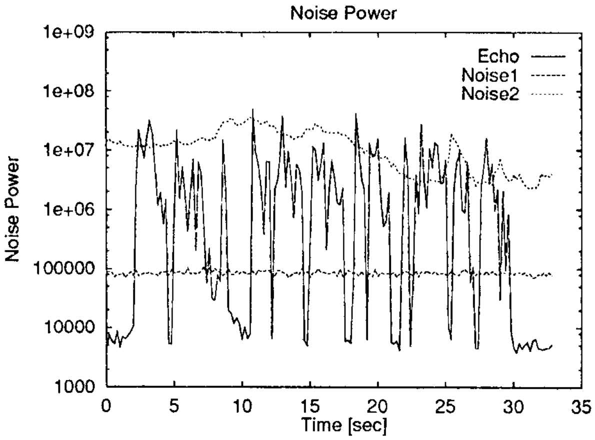

Fig. 6. Echo and Noise Power.

contains an engine noise, a wind noise and also brake noises, has also been used. Figure 6 shows the echo power and the noise power. Noise1 is a noise in an idle state and Noise2 is that in a moving state. The echo is larger than Noise1, while the ENR for Noise2 is almost always less than 0 dB.

In the first simulation, the proposed algorithm has been compared with the LMS algorithm, NLMS/Norm and NLMS/Leak. The parameters have been chosen as $\mu_{LMS} = 1.25 \times 10^{10}$ $\mu_{NLMS} = 0.1$, $a = 0.9999$, $a_s = 0.99$, $a_L = 0.99995$, $c_1 = 0.001$ and $c_2 = 0.05$. Note that the parameter $a = 0.99995$ makes NLMS/Leak unstable. Figure 3 compares the ERLE for Noise1. Thanks to the introduction of a lower limit on $P_X(t)$, the proposed algorithm is stable for large a_L and achieves superior echo reduction performance. From the beginning of the speech to 10 seconds, the ERLE of the proposed algorithm is larger than that of the others by almost 5dB.

The following simulations compare the proposed algorithm with three adaptive-step algorithms: the normalized stochastic gradient algorithm with a gradient adaptive and limited step-size (NSG-GALS)[5], the time-varying step-size NLMS (TVS-NLMS)[6], and an adaptive step-size based on the reference signal

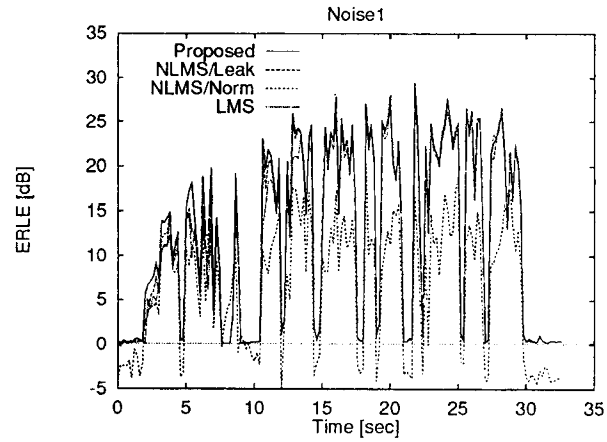

Fig. 3. Performance comparison with LMS and NLMS.

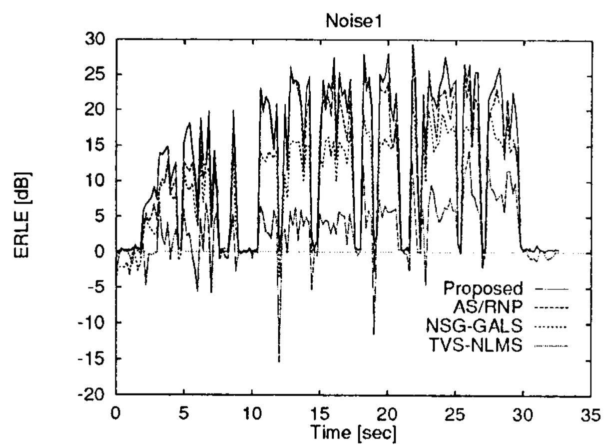

Fig. 4. Performance comparison with adaptive-step algorithms.

and the noise power (AS/RNP)[7]. The ERLE has been examined for both Noise1 and Noise2. The parameters have been optimized for Noise1 and have been chosen as $\mu_0 = 0.1$, $\rho = 1.0 \times 10^{-4}$, $\alpha = 2.0 \times 10^{-5}$ and $\beta = 0.0001$ for NSG-GALS, $\rho = 0.01$ and $\varepsilon = 1.0 \times 10^{6}$ for TVS-NLMS and $\mu_0 = 0.2$ and $\alpha = 50.0$ for AS/RNP.

Figure 4 compares the ERLE of those four algorithms for Noise1. The proposed algorithm converges faster than the conventinal algorithms. From 2 seconds to 10 seconds, the proposed algorithm achieves the ERLE almost 5dB larger than three adaptive-step algorithms.

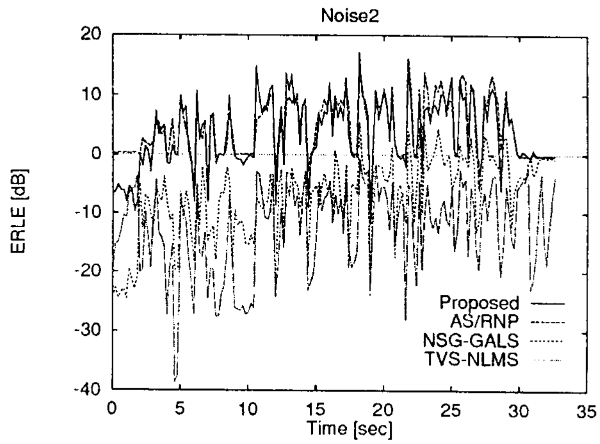

Fig. 5. ERLE for larger noise.

The final ERLE of the proposed algorithm is almost 30dB, which is comparable with AS/RNP and is more than 10dB larger than NSG-GALS and TVS-NLMS.

The ERLE for Noise2 is shown in Fig. 5. The proposed algorithm achieves more than 15dB of the ERLE. The ERLE is almost same as AS/RNP and more than 20dB larger than NSG-GALS and TVS-NLMS.

6. Conclusion

A modified NLMS algorithm based on a long-term average power has been proposed. The reference input signal power for normalization is estimated by using two leaky integrators with a short and a long time constants. A new power estimator in the proposed algorithm guarantees stability and enhances robustness against the additive noise. The amount of computation required for the proposed algorithm is almost same as the NLMS algorithm. Simulation results show that the proposed algorithm reduces echoes by almost 30dB in noisy environments. The convergence speed and the echo reduction performance are superior to some previously proposed adaptive-step algorithms.

Acknowledgments

The authors wish to thank the senior manager Dr. Yukitsuna Furuya and fellow reserachers of Personal C&C Research Laboratories, NEC Corporation, for providing us speech data used for computer simulations. Without these data, this work was not possible. They are also indebted to Dr. Takao Nishitani, the senior manager of Signal Processing Research Laboratory, Information Technology Research Laboratories, NEC Corporation for his guidance and continuous encouragement.

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