

# A Polyphase and FFT Realization of Modulation Sub-band Adaptive Filter with Minimum Sampling Rate

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## Abstract

A sub-band adaptive filter, in which modulation and demodulation are employed, were proposed [1],[2]. The sampling rate can be minimized, while no aliasing occurs. In this paper, a polyphase and FFT realization of this sub-band adaptive filter is proposed. In the polyphase and FFT realization for the transmultiplexer [3], the sampling rate reduction is the same as the number of the sub-bands. In the proposed method, however, they are different, so the conventional method cannot be applied. In the new realization, the polyphase filters are divided into a tapped delay line and an FFT part so as to be shared by all the sub-bands. Computational complexity is well reduced compared with the direct realization.

## 1.Introduction

Recently, multimedia technology has been developed, which make communication using both audio and image easy. In this case, if loud speakers are used, acoustic echo causes some problem. Audio echo cancelers are used for this reason. However, the impulse response length of room acoustic characteristics is very long. It requires several thousand samples with a sampling rate of 8 kHz. Therefore, a very high order adaptive filters are required, and at the same time, convergence speed becomes very slow. Therefore, sub-band adaptive filters become very important in order to save computational complexity and to make fast convergence possible[4],[5].

One problem in the sub-band adaptive filter is how to determine the sampling rate. Since the aliasing causes some distortion, the sampling rate must be higher than twice of the bandwidth of each sub-band. At the same time, computational complexity must be minimized. The modulation sub-band adaptive filter can satisfy these requirements [1],[2]. The sampling rate can be set to just the twice of the bandwidth of each sub-band. Let the number of sub-bands be  $M$ , and the sampling rate reduction be  $1/K$ , then  $M/K$  can be selected as any rational number. Additional requirement for computation is small.

On the other hand, the Polyphase and FFT realization of a filter-bank, which was applied to the TDM-FDM transmultiplexers [3], and the sub-band coding [6], is very efficient in order to reduce computational complexity of the filter bank. The filter bank can be realized

with the lower sampling rate  $f_s/K$ . However, in this case, the number of the sub-bands  $M$  and the sampling rate reduction  $1/K$  should be the same, that is  $M = K$ . On the other hand, in the modulation sub-band filter,  $M$  is not equal to  $K$ , rather  $M > K$ , and  $M/K$  is a rational number. Thus, the conventional polyphase and FFT realization cannot be applied to the modulation sub-band adaptive filter.

In this paper, a new method for polyphase and FFT realization is proposed. Computational complexity is also compared with the other methods.

## 2. Modulation Sub-band Adaptive Filter

Figure 1 shows a block diagram of the modulation sub-band adaptive filter [2]. The analysis filter bank consists of complex filters  $F_i(z)$ , the synthesis filter bank consists of complex filters  $G_i(z)$ . The number of sub-band is  $M$  and the sampling rate reduction is  $1/K$ . The carrier signals applied to both modulator (MOD) and demodulator (DEM) are complex. The input signal  $x(n)$  is real. In the analysis filter bank and the modulator, the signals are represented with complex number. After the modulators, only the real part is transferred. The adaptive filters (AF) are real filters. They are adjusted using the error at each adaptive filter output. In the demodulator and the synthesis filter bank, the signals are also represented with complex number. After the synthesis filter bank, only the real part is transferred.

Figure 2 shows examples of the signal spectra at the analysis filter bank and the modulator. The number of sub-bands  $M$  is 6. The frequency of 1 means the sampling rate  $f_s$ . The output of  $F_i(z)$ ,  $i = 0 \sim 5$ , are shown in Fig.2(a), and one of them, that is the  $F_1(z)$  output, is shown in Fig.2(b). This spectrum is shifted to the origin by the modulator as shown in Fig.2(c). By taking the real part of the modulator output, the symmetry part appears as shown in Fig.2(d). Finally, this real signal is down sampled by  $f_s/K$ ,  $K = 4$ , as shown in Fig.2(e). The spectrum is expanded over a whole band  $0 \leq f \leq f_s/K$ . However, aliasing does not occur.  $K$  is not the same as  $M$ . The signal spectra in the demodulator and the synthesis filter bank are the same in the reversed order, and explanation is omitted here.

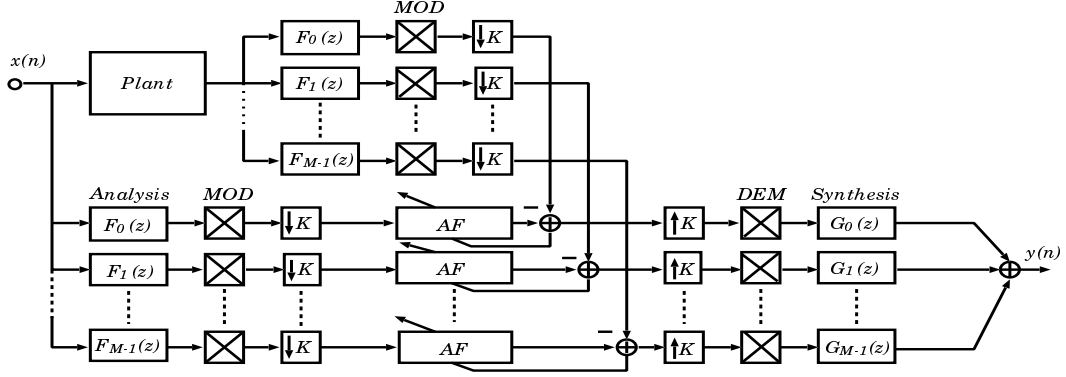


Fig.1 Modulation sub-band adaptive filter.

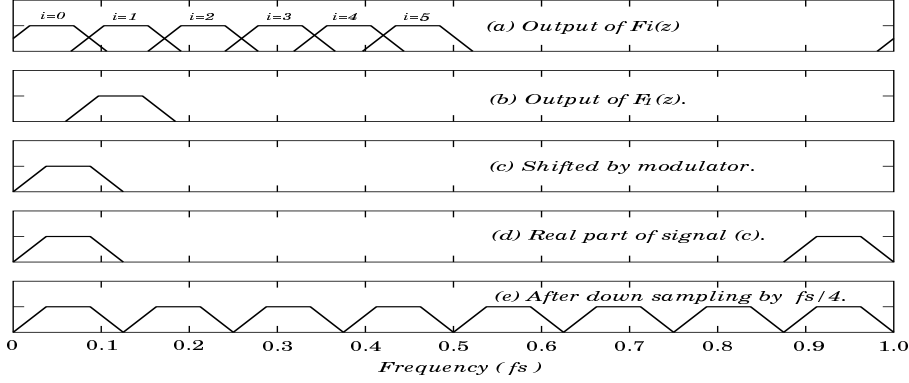


Fig.2 Signal spectra at analysis filter bank and modulator outputs.

### 3. Polyphase and FFT Realization

#### 3.1 Modulation Sub-band Adaptive Filter

The analysis filter bank is taken into account for mathematical explanation here. The same derivation is possible for the synthesis filter bank.

The fundamental low-pass filter (LPF) is denoted

$$F(z), \quad z = \exp(j2\pi f/f_s) \quad (1)$$

$f_s$  is the sampling rate.  $F_i(z)$  are obtained by shifting  $F(z)$  as follows:

$$F_i(z) = F(z_i), \quad i = 0 \sim M - 1 \quad (2)$$

$$z_i = \exp\left[\frac{j2\pi(f - f_s/4M - if_s/2M)}{f_s}\right] \quad (3)$$

If the transient bandwidth is given by  $2\Delta f$ , then a single sub-band occupies  $f_s/2M + 2\Delta f$ . Thus, the minimum sampling rate after down sampling is

$$f_{st} = 2(f_s/2M + 2\Delta f). \quad (4)$$

Furthermore, the following relations are held.

$$f_{st} = f_s/K \quad (5)$$

$$\Delta f = \frac{f_s}{4} \left( \frac{1}{K} - \frac{1}{M} \right) \quad (6)$$

Next, the carrier signal applied to the modulator is given by

$$c_i(n) = \exp\left[\frac{j2\pi(-\Delta f + if_s/2M)n}{f_s}\right] \quad (7)$$

#### 3.2 Polyphase Filter Realization

Equation (3) is modified as follows:

$$z_i = \exp\left[\frac{j2\pi(f - f_s/4M - if_s/M)}{f_s}\right] \quad (8)$$

The shifting by  $if_s/2M$  is replaced by  $if_s/M$ , which can provide the same spectrum arrangement as shown in Fig.2 [3]. However the relation between  $F_i(z)$  and the corresponding sub-band is changed. The arrangement  $(F_0(z), F_1(z), F_2(z), F_3(z), F_4(z), F_5(z))$  is changed to  $(F_0(z), F_5(z), F_1(z), F_4(z), F_2(z), F_3(z))$ . This is not a practical problem. In this paper, FIR filters are used for  $F(z)$ . However, the following discussion can be applied to IIR filters in the same way.

$F(z)$  is expanded as follows:

$$F(z) = F_{i,0}(z^K) + z^{-1}F_{i,1}(z^K) + \dots + z^{-(K-1)}F_{i,K-1}(z^K) \quad (9)$$

The frequency shifting in Eq.(8) becomes

$$z_i^K = \exp\left[\frac{j2\pi K(f - f_s/4M - if_s/M)}{f_s}\right]$$

$$\begin{aligned}
&= \exp\left(\frac{j2\pi K f}{f_s}\right) \exp\left(\frac{-j2\pi K}{4M}\right) \exp\left(\frac{-j2\pi i K}{M}\right) \\
&= z^K w^{-1/4} w^{-i}, \\
&w = \exp(j2\pi K/M)
\end{aligned} \tag{10}$$

$$\begin{aligned}
z_i^{-j} &= z^{-j} v^{j/4} v^{ij}, \quad v = \exp(j2\pi/M), \\
j &= 0 \sim K-1
\end{aligned} \tag{11}$$

Substituting Eqs.(10) and (11) into Eqs.(2) and (9),  $F_i(z)$  is expressed as

$$\begin{aligned}
F_i(z) &= F(z_i) = \sum_{j=0}^{K-1} z_i^{-j} \tilde{F}_{l,j}(z_i^K) \\
&= \sum_{j=0}^{K-1} z^{-j} v^{j/4} v^{ij} \tilde{F}_{l,j}(z^K w^{-1/4} w^{-i})
\end{aligned} \tag{12}$$

Here the following notation will be used.

$$F_{i,l,j}(z^K) = \tilde{F}_{l,j}(z^K w^{-1/4} w^{-i}) \tag{13}$$

Figure 3 shows a block diagram to implement Eq.(12). The term  $z^{-j} v^{j/4}$  can be commonly used by all  $F_i(z)$ ,  $i = 0 \sim M-1$ . However, the other term  $v^{ij} \tilde{F}_{l,j}(z^K w^{-1/4} w^{-i})$  are different for each sub-band. How to simplify this part is the next problem.

### 3.3 Polyphase & FFT Realization

In Eq.(12),  $w^{1/4}$  is common for all  $F_{i,l,j}(z^K)$ , then it can be combined with their coefficients. The other term  $w^i = \exp(j2\pi i K/M)$  is different from the conventional filter bank [3]. If  $K = M$ , then this term is unity, and becomes the same for all  $F_i(z)$ . This means that  $F_{i,l,j}(z^K)$  are common for all  $F_i(z)$ . In this case,  $v^{ij}$  can be realized as the discrete Fourier transform (DFT), and if  $M$  equals to the power of 2, DFT is implemented by FFT [3].

However, when the number of sub-bands  $M$  is not equal to the sampling rate reduction  $K$ ,  $w^i$  remains. How to implement this term with less additional computation is described in the following.

The sub-filters  $F_{i,l,j}(z^{-K})$  is expressed as

$$\begin{aligned}
F_{i,l,j}(z^K) &= h_{l,j,0} + h_{l,j,1} w^i z^{-K} + h_{l,j,2} w^{2i} z^{-2K} \\
&\quad + \dots + h_{l,j,P-1} w^{(P-1)i} z^{-(P-1)K} \\
&= \sum_{p=0}^{P-1} h_{l,j,p} w^{pi} z^{-pK}
\end{aligned} \tag{14}$$

where,  $h_{l,j,p}$  already includes  $w^{p/4}$ . Figure 4 shows a block diagram of  $F_{i,l,j}(z^K)$ .  $w^{pi}$  are separated from  $h_{l,j,p}$ . Letting the output of  $h_{l,j,p} z^{-pK}$  be  $u(j,p)$ , the output of  $F_i(z)$  becomes

$$\sum_{p=0}^{P-1} w^{pi} u(j,p) \tag{15}$$

From this figure and Eq.(14), if  $P$  is equal to  $M$ , then the term  $w^{pi} = \exp(j2\pi pi K/M)$  is equivalent to DFT,

of course the kernel, that is  $w = \exp(j2\pi K/M)$  is different from that of the original DFT, whose kernel is  $w = \exp(j2\pi/M)$ . However, if  $M$  is the power of 2, FFT can be applied to the former in the same way.

As shown in Fig.4,  $\tilde{F}_{l,j}(z^K)$  can be shared by all  $F_i(z)$ ,  $i = 0 \sim M-1$ . This means the same  $\tilde{F}_{l,j}(z^K)$  can be used by  $F_i(z)$ . Equation(15) can be implemented using FFT by combining all  $F_i(z)$ . Combining the block diagrams shown in Figs.3 and 4, a whole block diagram of the analysis filter bank, whose channel filters are expressed by Eq.(16), is composed as shown in Fig.5.

$$F_i(z) = \sum_{j=0}^{K-1} z^{-j} v^{j/4} \left\{ \sum_{p=0}^{P-1} h_{l,j,p} w^{pi} z^{-pK} \right\} v^{ij} \tag{16}$$

In this realization, however, the term  $v^{i,j}$  cannot be realized by DFT. Another point is the condition  $P = M$  required above. This condition can be relaxed into  $P = nM$ ,  $n$  is an integer. For example, when  $P = 2M$ , Eq.(15) can be rewritten as follows:

$$\sum_{p=0}^{P-1} w^{pi} u(p,i) = \sum_{p=0}^{M-1} w^{pi} [u(p,i) + u(p+M,i)] \tag{17}$$

$$w^{(p+M)i} = w^{pi}, \quad w = \exp(j2\pi K/M) \tag{18}$$

Let the filter length of the fundamental LPF  $F(z)$  be  $N$ . Since  $P = N/K$ , the condition  $P = nM$  can be replaced by

$$N = nMK \tag{19}$$

For example,  $M = 8$ ,  $K = 6$ , then  $N = 48n$ , that is  $N = 48, 96, 144$ .  $N$  is required to be selected around these numbers.

### 3.4 Computational Complexity

Computational complexity of the analysis filter bank is evaluated here. In Fig.5,  $v^{j/4}$  requires  $2K$  real multiplications at the sampling rate  $f_s/K$ .  $\tilde{F}_{l,j}(z^K)$  requires  $4N$  real multiplications at  $f_s/K$ , FFT requires  $2KM \log_2 M$  real multiplications at  $f_s/K$ . Finally,  $v^{ij}$  needs  $4KM$  real multiplications at  $f_s/K$ . Totally, the analysis filter bank needs the following real multiplications at  $f_s$ .

$$Q_1 = 2 + 4N/K + 2M \log_2 M + 4M \tag{20}$$

On the other hand, the conventional polyphase & FFT method, in which  $K = M$ , requires

$$Q_2 = 2 + 4N/M + 2 \log_2 M \tag{21}$$

Furthermore, the direct realization at  $f_s$  as shown in Fig.1 requires

$$Q_3 = NM \tag{22}$$

(Examples)

(1)  $N = 48$ ,  $M = 8$ ,  $K = 6$ , then  $Q_1 = 114$ ,  $Q_2 = 32$  and  $Q_3 = 384$ .

(2)  $N = 96$ ,  $M = 8$ ,  $K = 6$ , then  $Q_1 = 146$ ,  $Q_2 = 56$  and  $Q_3 = 768$ .

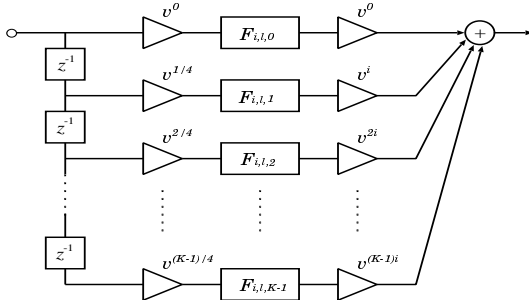


Fig.3 Block diagram of  $F_i(z)$  expressed by Eq.(12).

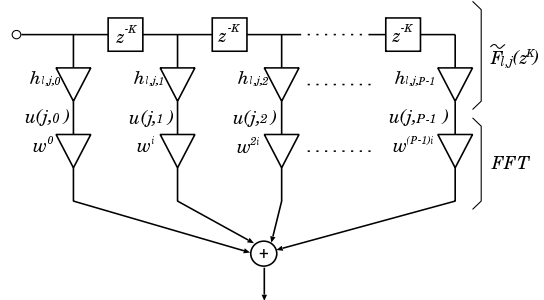


Fig.4 Block diagram of  $F_{i,l,j}(z^K)$  expressed by Eq.(14).

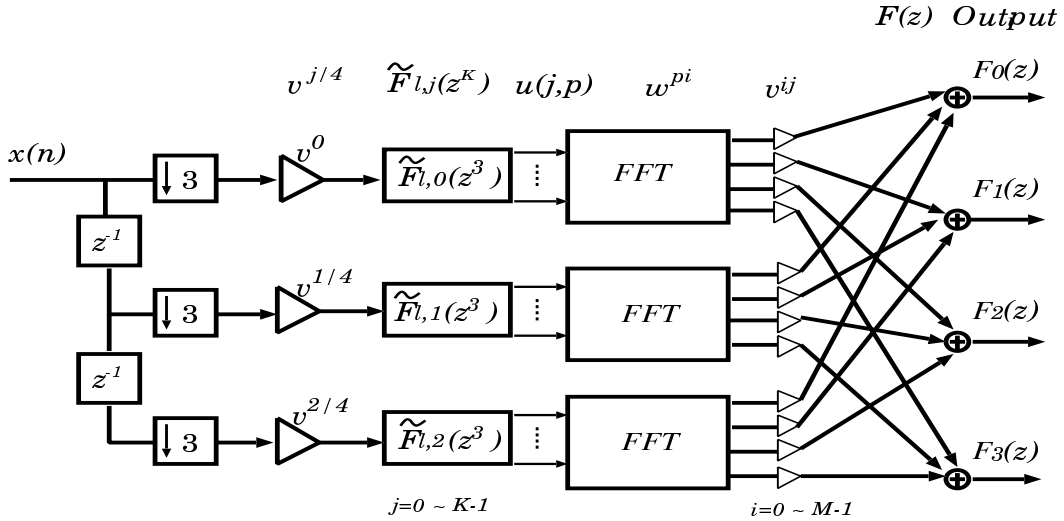


Fig.5 Proposed polyphase & FFT realization of analysis filter bank.

The proposed method requires more computations than the conventional polyphase & FFT method with  $K = M$ . However, the latter model cannot be used in the sub-band adaptive filters due to aliasing. The proposed can save computations drastically compared with the direct realization shown in Fig.1.

## 4. Simulation

The sub-band adaptive filter, using the analysis filter bank shown in Fig.5, has exactly the same characteristics as that of the original shown in Fig.1.

## 5. Conclusions

A polyphase & FFT realization, which is different from that for the transmultiplexer, is proposed for the modulation sub-band adaptive filter. The polyphase filters are different for the sub-bands due to mismatch between  $M$  and  $K$ . Therefore, the polyphase filters are divided into a transversal filter and an FFT part so as to be shared by all the sub-bands. Computational complexity can be well reduced compared with the direct realization.

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