PERMUTED DIFFERENCE COEFFICIENT DIGITAL FILTERS

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ABSTRACT

A new difference coefficient FIR digital filter is proposed in this paper. Its coefficient is obtained as the difference between the successive values of the original coefficients permuted in large magnitude sequence. It is effectively applied to every kinds of filter responses. Computation complexity becomes about 18 % and 13% for 99th and 299th order FIR filters, compared with a direct FIR filter realization.

INTRODUCTION

FIR filters are suitable for a linear phase and a stable response realization. A high order filter, however, is required to realize a high Q frequency response. It is necessary to perform a large number of multiplier operations in a sampling period. Then decreased or simplified multiplier operations become very important for a high order FIR filter realization. Several kinds of approaches to implement multiplier operations with lower computation rate such as distributed arithmetics (1), (2), (3) and residue number systems (4) have been proposed. Difference routing digital filter proposed by Gerwen et al can reduce multiplier operations for a narrow band and low Q filter (5).

In this paper a new difference coefficient FIR digital filter is proposed. Its coefficient is obtained as the difference between successive values of the original coefficients which are permuted in large magnitude sequence. The proposed digital filter is called Permuted Difference Coefficient Digital Filter (PDC-DF) in this paper. It can be effectively applied to every kinds of filter characteristics. The PDC-DF algorithm, computation complexity and a hardware realization are described.

PDC-DF ALGORITHM

The original filter coefficient h_n is permuted in sequence of large magnitude at first. New difference coefficient is obtained as the difference between the successive values of the reordered coefficients. This process is repeated, and higher order difference coefficients can be obtained.

Input and output signal x(n) and y(n) satisfy the following convolution equation in FIR filters

$$y(n) = \sum_{m=0}^{N-1} h_m\, x(n-m) \tag{1}$$

where n is assumed to be larger than N-1. Let h_k^* be an absolute value of h_m, and k be the reordered index, then h_k^* satisfies the following conditions

$$h_k^* = |hm| \tag{2a}$$

and

$$0 \leq h_0^* \leq h_1^* \leq h_2^* \cdots \leq h_{N-1}^*. \tag{2b}$$

Let $x^*(n-k)$ be $\text{sign}(h_m)x(n-m)$, then y(n) is expressed using h_k^* and $x^*(n-k)$ as follows:

$$y(n) = \sum_{k=0}^{N-1} h_k^*\, x^*(n-k). \tag{3}$$

First-order permuted difference coefficient $\Delta_k(1)$ for h_k^* is defined as follows:

$$\Delta_k(1) = h_k^* - h_{k-1}^*,\ k = 1, 2, \cdots, N-1 \tag{4a}$$

$$\Delta_0(1) = h_0^*. \tag{4b}$$

Using $\Delta_k(1)$, y(n) is rewritten as follows;

$$y(n) = \sum_{k=0}^{N-1} \Delta_k(1) \cdot u^{(1)}(n-k) \tag{5}$$

where

$$u^{(1)}(n-k) = \sum_{i=k}^{N-1} x^*(n-i). \tag{6}$$

$u^{(1)}(n-k)$ can be calculated through the following accumulation

$$u^{(1)}(n-k) = u^{(1)}(n-k-1) + x^*(n-k),$$

$$k = 0, 1, \cdots, N-2 \tag{7a}$$

$$u^{(1)}(n-N+1) = x^*(n-N+1). \tag{7b}$$

Number of additions in Eq. (7) is N-1.

Second-order permuted difference coefficient can be obtained in a similar way as the 1st-order difference ocefficient. Let $\Delta_\ell^*(1)$ be $\Delta_k(1)$, and ℓ be the reordered index, that is,

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$$\Delta_\ell^*(1) = \Delta_k(1) \quad (8a)$$

and

$$\Delta_0^*(1) \leqq \Delta_1^*(1) \leqq \Delta_2^*(2) \cdots \leqq \Delta_{N-1}^*(1). \quad (8b)$$

The 2nd-order difference coefficient $\Delta_\ell(2)$ is defined by Eq. (9)

$$\Delta_\ell^{(2)} = \Delta_\ell^*(1) - \Delta_{\ell-1}^*(1), \ \ell = 1, 2, \cdot\cdot, N-1 \quad (9a)$$

$$\Delta_0^{(2)} = \Delta_0^*(1). \quad (9b)$$

Using $\Delta_\ell^{(2)}$ y(n) becomes

$$y(n) = \sum_{\ell=0}^{N-1} \Delta_\ell^{(2)} \cdot u^{(2)}(n-\ell)$$

$u^{(2)}(n-\ell)$ is expressed by the reordered $u^{(1)}(n-k)$ represented as $u*^{(1)}(n-\ell)$ which corresponds to $\Delta_\ell^*(1)$.

$$u^{(2)}(n-\ell) = \sum_{i=\ell}^{N-1} u*^{(1)}(n-i). \quad (11)$$

Equation (11) can be performed through N-1 times additions

$$u^{(2)}(n-\ell) = u^{(2)}(n-\ell-1) + u*^{(1)}(n-\ell),$$

$$\ell = 0, 1, \cdot\cdot, N-2 \quad (12a)$$

$$u^{(2)}(n-N+1) = u*^{(1)}(n-N+1). \quad (12b)$$

Higher order difference coefficient can be obtained in the same way.

COMPUTATION COMPLEXITY

Additions are required for $u^{(1)}(n-k)$ and $u^{(2)}(n-\ell)$ calculations given by Eqs. (7) and (12), and mutiplications are required for the $\Delta_\ell(2)$ and $u^{(2)}(n-\ell)$ product calculation. Let N_1 and N_2 be the number of nonzero $\Delta_k^{(1)}$ and $\Delta_\ell^{(2)}$ rounded off, and L_{PDC} be the $\Delta_\ell(2)$ word lengths reduced by $\log_2(\max|h_n|/\max(\Delta_\ell(2)))$ bit from L_0. The computation rate for the PDC-DF is defined as follows:

$$I_{PDC} = N + N_1 + L_{PDC} \cdot N_2. \quad (13)$$

I_{PDC} corresponds to number of equivalent additions. The direct FIR structure is evaluated in the same way. Its computation complexity I_{FIR} becomes

$$I_{FIR} = (L_0 + 1)N. \quad (14)$$

L_0 which is the h_n word length, is determined so as to realize the desired filter response. Relation between the factors, N_1, N_2 and L_{PDC} determining computation complexity and filter response can be found based on the ideal filter response. The $\max|h_n|$, the mean and the variance for h_n ($E[h_n]$ and $Var[h_n]$) can be expressed using band-width B and number of coefficients N as follows:

$$\max_n |h_n| = 2B \quad (15a)$$

$$E[h_n] = \frac{1}{N} \text{ Low pass filter}$$

$$= 0 \text{ Band pass filter} \quad (15b)$$

$$Var[h_n] = \frac{2B}{N} - \frac{1}{N^2} \cong \frac{2B}{N} \quad (15c)$$

Standard variance for h_n is $\sqrt{Var[h_n]} = \sqrt{2B/N}$. The distribution function property can be evaluated by the ratio of the standard variance and the maximum value, which is $1/\sqrt{2BN}$.

Narrow band filters

Since the ratio $1/\sqrt{2BN}$ is large, the probability desnity for large magnitude coefficients is high. The max $(\Delta_k(1))$ is mainly determined as the difference among large magnitude coefficients. Thus it is greatly reduced from the $\max|h_n|$. On the other hand, N_1 is not so small compared with N because of high probability density for large magnitude coefficients. The same properties are roughly held for the $\max(\Delta_\ell^{(2)})$ and N_2.

Wide band filters

The ratio $1/\sqrt{2BN}$ is small, then the probability density for small magnitude coefficients is high. The $\max(\Delta_\ell(2))$ cannot be well decreased because of low probability density for large magnitude coefficients. On the other hand N_1 and N_2 are well decreased from high probability density for small magnitude coefficients. The computation complexity is determined by two kinds of factors, $\max(\Delta_\ell^{(2)})$ and the number of nonzero coefficients, then the above two kinds of filter responses have almost the same computation complexity.

Filter Order

Another factor determining the coefficient distribution function is a filter order N-1. Since the ratio $1/\sqrt{2BN}$ becomes small in high order FIR filters, N_1 and N_2 can be decreased. On the other hand, the $\max(\Delta_\ell^{(2)})$ is not so increased compared with low order filters. The reason can be explained as follows: Under the same probability density, high order filters can give smaller magnitude difference coefficients, compared with low degree filters. Then, high order filters provide almost the same value for $\max(\Delta_\ell(2))$ as low order filters, in spite of the lower probability density for large magnitude coefficients. As a result, the computation complexity for high order FIR filters can be reduced compared with that for low order filters.

NUMERICAL EXAMPLE

Numerical examples showing the coefficient distribution nature and the computation complexity for several kinds of filter responses are discribed here. The FIR filters used in the following discussions are as follows: The 99th and 299th order low pass filters (LPF) and band pass filters (BPF) designed through the Remetz exchange method (6).

Examples for thier frequency and the time responses are shown in Fig. 1. The bandwith B is determined by the 6dB loss point shown with fc in the figure.

Coefficient Distribution Function

Figure 2 shows provability densities for $|h_n|$, $\Delta_k^{(1)}$ and $\Delta_\ell^{(2)}$. One division indicates 10 % and 1 bit on the vertical and the horizontal axes, respectively. All coefficients are normalized by the $\max|h_n|$ and illustrated by its word lengths. The probability density for (i-1, i] bit on the horizontal axis corresponds to the number of the coefficients whose absolute value is included in the following region: $(2^{-L_0+i-1}, 2^{-L_0+i}]$. L_0 is taken as 10 bits for the 99th filter and 12 bits for the 299th filter. From these figures, the previously obtained distribution function natures can be confirmed.

Computation Complexity

The $\max(\Delta_\ell^{(2)})$, N_1 and N_2 are shown in Fig. 3. N_1 and N_2 are normalized by N. Since, the $\max(\Delta_\ell^{(2)})$ is not normalized, there is about a 2 bit difference between the 99th and the 299th filters. This means that word length reductions for $\max(\Delta_\ell^{(2)})$ in both the 99th and the 299th filters are almost the same. From this fact, the previosuly obtained conjecture, that is, the $\max(\Delta_\ell^{(2)})$ is independent on filter degree, can be recognized. The discussed relation between the complexity factors, and the bandwidth B and filter order N-1 is also confirmed by these results. The computation complexity evaluated by Eq. (13) is illustrated in Fig. 4, where I_{PDC} is normalized by I_{FIR}. They are around 18 % and 13 % for the 99th and 299th filters, respectively, compared with the direct method. As previsouly discussed the high order filters can give small computation rate. The independency between I_{PDC} and bandwidth is also recognized from this figure.

HARDWARE REALIZATION

Through the previous numberical examples, the 2nd-order PDC-DF can be found to be useful from the computation complexity point of view. Therefore a hardware realization is described for the 2nd-order PDC-DF here. For the sake of clarifying the PDC-DF algorithm, 9th order filter is used in the following. The original coefficient, the 1st and the 2nd order difference coefficient are shown in Table 1. A hardware realization is illustrated in Fig. 5. In this example, the original coefficient (L_0) and 2nd-order difference coefficient (L_{PDC}) word lengths are 9 bits and 1 bits, respectively. The number of nonzero difference coefficient N_1 and N_2 are 6 and 2, respectively. The data mapping block DM can be easily realized using a random access memory (RAM) together with a read only memory (ROM) where address signal is stored.

CONCLUSION

A new difference coefficient digital filter is proposed. It can be effectively applied to every kind of filter responses. Computation complexities are around 18 % and 13 % for 99th and 299th FIR filters, respectively, compared with a direct FIR filter realization.

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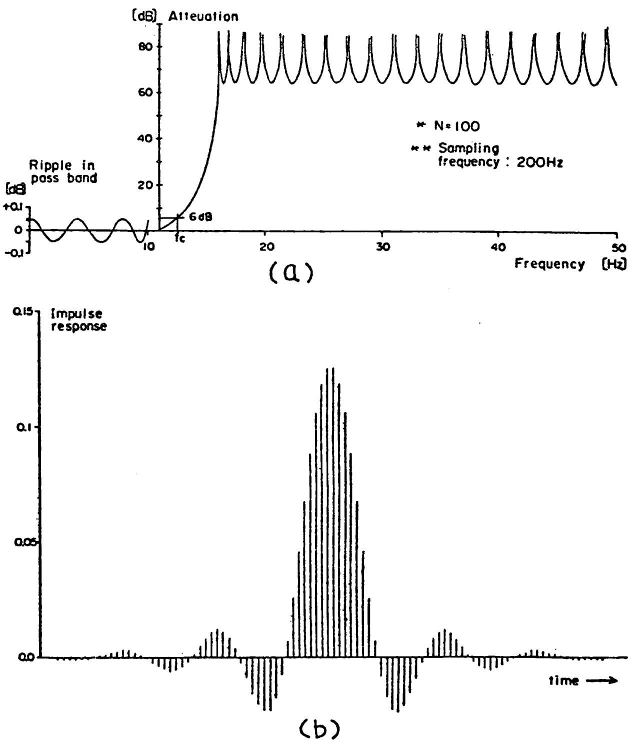

Fig. 1 Example for filter response used for computation complexity investigation. (a) Loss characteristics. (b) Impulse response.

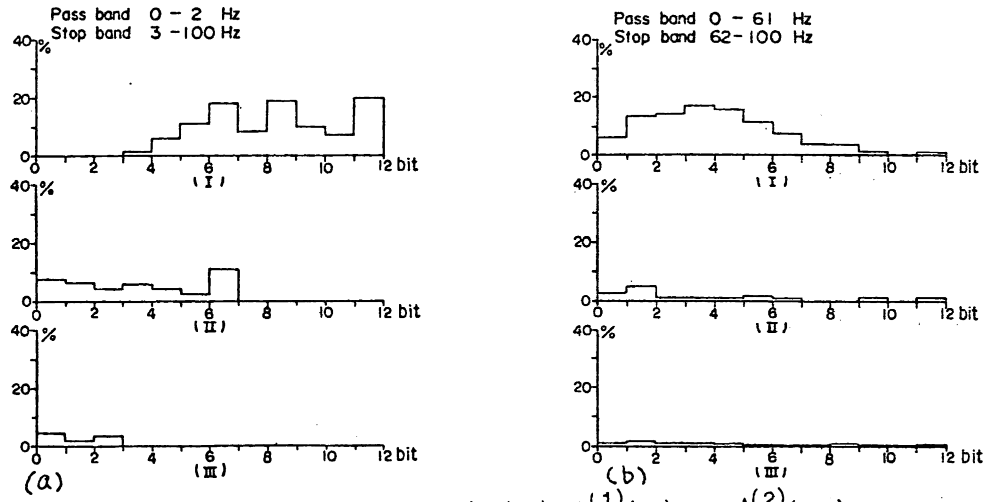

Fig. 2 Numerical examples for $|h_n|$(I), $\Delta_k^{(1)}$(II) and $\Delta_\ell^{(2)}$(III) distribution function. One segregation means 10 % and 1 bit on vertical and horizontal axes. 299th order LPF is used.

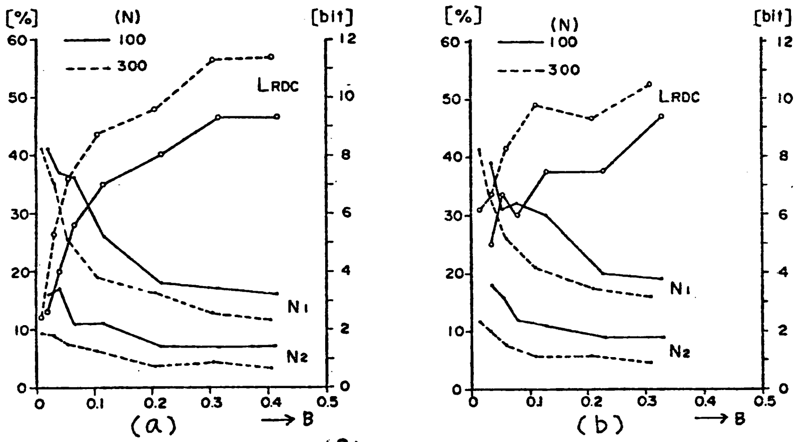

Fig. 3 Numerical examples for $\Delta_\ell^{(2)}$ word lengths L_{PDC}, N_1 and N_2. N_1 and N_2 are normalized by N and expressed in percentage. L_{PDC} are obtained using 10 and 12 bits L_0 for N=100 and 300, respectively, and not normalized. (a) Low pass filter. (b) Band pass filter.

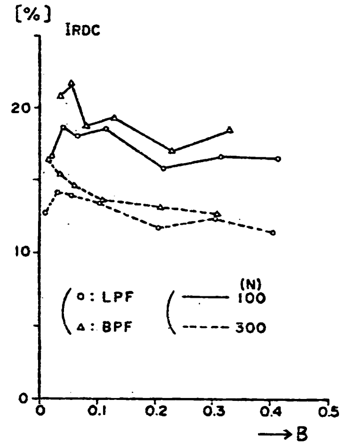

Fig. 4 Numerical examples for computation complexity I_{PDC} normalized by I_{FIR}.

Table 1 Example for permuted difference coefficients.

m	hm	k	h_k^*	$\Delta_k^{(1)}$		$\Delta_\ell^{*(1)}$	$\Delta_\ell^{(2)}$	j	$\Delta_j^{*(2)}$
0	0.25	0	0.0	0.0	0	0.0	0.0	0	0.0
1	-0.625	1	0.25	0.25	1	0.0	0.0	1	0.0
2	-0.375	2	0.375	0.125	2	0.0	0.0	2	0.0
3	0.875	3	0.375	0.0	3	0.0	0.0	3	0.0
4	0.375	4	0.5	0.125	4	0.125	0.125	4	0.0
5	0.5	5	0.625	0.125	5	0.125	0.0	5	0.0
6	0.0	6	0.625	0.0	6	0.125	0.0	6	0.0
7	-0.875	7	0.875	0.25	7	0.125	0.0	7	0.0
8	0.625	8	0.875	0.0	8	0.25	0.125	8	0.125
9	1.0	9	1.0	0.125	9	0.25	0.0	9	0.125

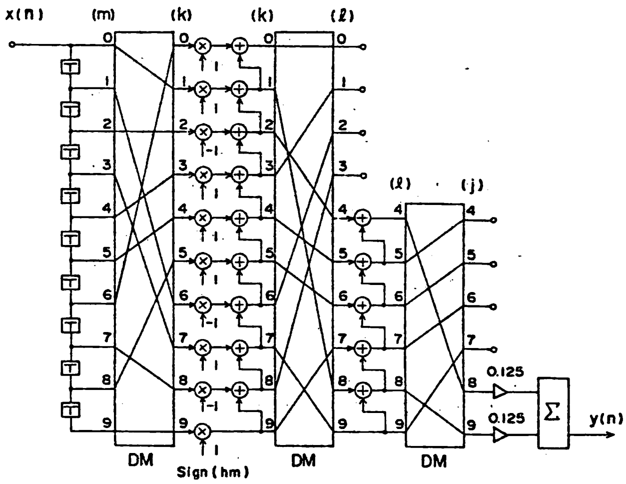

Fig. 5 Second-order PDC-DF hardware realization for 9th order FIR filter.