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OPTIMUM PAIRING AND ORDERING CONDITIONS FOR SIMULTANEOUS REDUCTION IN TOTAL CAPACITANCE, SENSITIVITY AND OUTPUT NOISE OF CASCADE SC FILTERS

Zhiqiang MA †*, *Kenji NAKAYAMA* † *and Gaishi YAMAMOTO* ‡

† Department of Electrical and Computer Engineering,
Faculty of Technology, Kanazawa University, Kanazawa, 920 Japan.
‡ Faculty of Engineering, Kanazawa Institute of Technology, Ishikawa, 921 Japan.

Abstract– This paper presents optimum pairing and ordering (P/O) conditions for simultaneous reduction in total capacitance, sensitivity and output noise of cascade SC filters. First, investigating relations among conditions proposed for minimizing these three parameters independently, termed, $C_{T\min}$, $S_{\min}$ and $N_{\min}$ conditions, we make it clear that there is no P/O assignment with which the three parameters can be simultaneously minimized. Then we verify that $N_{\min}$ condition is valid in reducing two other parameters. Trade-off optimum conditions are proposed by combining $N_{\min}$ with $S_{\min}$ and $C_{T\min}$. These conditions can limit the number of P/O assignments, in which an optimum P/O assignment for simultaneous reduction of three parameters can be searched out easily. Depending upon applications, the trade-off optimum condition can be slightly modified by adjusting the proportion of $C_{T_{\min}}$, $S_{\min}$ and $N_{\min}$. Efficiency of the proposed trade-off optimum conditions is confirmed through computer simulation using several kinds of filters.

1. INTRODUCTION

Cascade Switched-Capacitor Filters (SCFs) play an important role in practical applications, due to their flexible characteristics and simple circuit configuration. In cascade SCFs, the total capacitance, sensitivity and output noise, called three parameters in this paper, are highly dependent on pole-zero pairing and ordering (P/O). It is very important to reduce three parameters to obtain low power dissipation, low output noise, low sensitivity, high operating speed, and a small chip area in monolithic integrated SCFs.

Effects of P/O on three parameters have been well studied. Optimum P/O conditions for minimizing each parameter independently have been proposed [1]-[3]. In practical applications, however, it is desirable to find a P/O condition which can simultaneously minimize three parameters. The question arises on whether there exists this kind of P/O condition. If there is no such a P/O condition, the question is changed into the following. How can three parameters be simultaneously reduced?

In this paper, first, it is verified that there exists no such a P/O assignment with which three parameters can be simultaneously minimized. Second, combined conditions are proposed which can simultaneously reduce three parameters, taking optimum trade-off among three parameters into account.

2. SEPARATE P/O CONDITIONS FOR MINIMIZING TOTAL CAPACITANCE, SENSITIVITY AND OUTPUT NOISE

2.1. Biquad SC circuit

In our discussion, the biquad SC circuit in Fig.1 is considered [4]. The resulting P/O conditions are valid for other biquad circuit configurations, such as the one proposed by Fleischer & Laker[5].

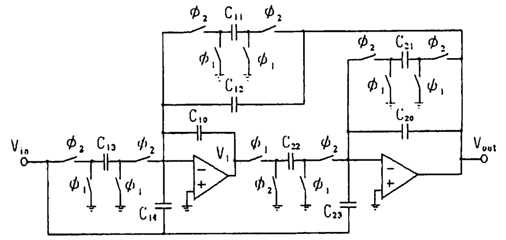

Fig.1 Biquad SC circuit by Martin & Sedra.

2.2. Optimum P/O conditions

As shown in our previous papers [1]-[3], optimum P/O conditions have been proposed for minimizing the total capacitance, sensitivity and output noise independently. These P/O conditions are called $C_{T\min}$, $S_{\min}$ and $N_{\min}$ conditions. they are summarized below.

$C_{T\min}$ condition:

Zeros close to high-Q poles are assigned to the preceding sections of high-Q pole sections.

High-Q poles are preceded by Low-Q poles.

$S_{\min}$ condition:

Pairing: High-Q poles are paired with zeros close to them.

Ordering: Sensitivity is not affected by ordering.

$N_{\min}$ condition:

Pairing: A pole is paired with the nearest zero in descending order of pole Q.

Ordering: Biquad circuits are so arranged that their amplitude peak frequencies are dispersed.

3. DISTRIBUTIONS OF P/O ASSIGNMENTS WITHOUT ANY CONSTRAINTS

In the conditions shown in the previous section, the pairing of the $N_{\min}$ condition is one of pairings satisfying the $S_{\min}$ condition. Furthermore, in the $N_{\min}$ condition, the ordering is much dominant [2]. Therefore, by using the pairing of the $S_{\min}$ condition and the ordering of the $N_{\min}$ condition, the sensitivity and output noise can be simultaneously minimized. On the contrary, the paring of the $C_{T\min}$ condition conflicts with both $S_{\min}$ and $N_{\min}$ conditions. Therefore, it is impossible to minimize the total capacitance using the pairing, with which the sensitivity and the output noise are at their minimums. This means that there exists no P/O assignment with which three parameters can be simultaneously minimized. This analytical result was examined through computer simulation using several kinds of filters. One of them is shown here.

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Filter characteristics and parameters used in this simulation are shown in Table 1. Poles and zeros are shown in Fig.2. The distributions of three parameters without any constraints on P/O are shown in Fig.3. In this figure, a symbol Δ corresponds to a single P/O assignment. C_T and N express total capacitance and output noise, respectively. $C_{T\min}$ and $N_{\min}$ represent their minimum values. Sensitivity was evaluated by using a huge amount of capacitance deviation patterns [3]. The rate of capacitance deviation patterns, with which amplitude deviations at the cutoff frequency are within 10%, are shown in percentage in Fig.3. The minimum sensitivity corresponds to the maximum rate of capacitance deviation patterns, which is 35.8%.

In Fig.3(a) and (b) there are no P/O assignments nearer by the origin. This means that there is no P/O assignment which can simultaneously minimize three parameters. Therefore, it is desirable to find a trade-off optimum assignment, which can make two parameters to be as small as possible, while maintaining the third parameter to be at the minimum. In high-order filters, the number of all possible P/O assignments is very large. Furthermore, the number of trade-off optimum assignments is small. Therefore, a lot of computing time is required to search for a trade-off optimum assignment. In order to save computing time, trade-off optimum conditions are required to select limited candidates for an optimum P/O assignment, with which three parameters can be simultaneously reduced.

Tab.1. Filter characteristics and parameters

Pass-band ripple	0.177dB
Stop-band loss	80.8dB
Cut-off frequency	$0.1f_s$
Sampling frequency f_s	40kHz
Clock	50% duty
On resistance	10kΩ
Temperature	25°C
Noise in OpAmp	white noise: 50nv/Hz
(Unity gain band =4MHz)	1/f noise: 50nv/Hz at 1kHz
unit capacitance	0.4pF

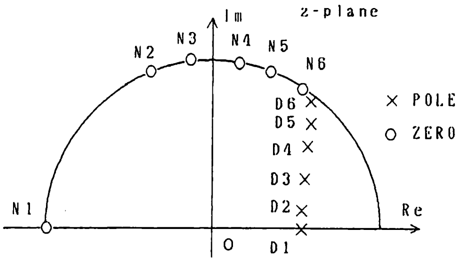


Fig.2 Location of poles and zeros.

4. EFFECTS OF SEPARATE CONDITIONS ON THREE PARAMETERS

4.1. Efficiency of $N_{\min}$ condition

By using the $C_{\min}$, $S_{\min}$ and $N_{\min}$ conditions separately, their contributions to three parameters were investigated. The results show that only the $N_{\min}$ condition is valid for reducing two other parameters. The distributions of three parameters with the $N_{\min}$ condition are shown in Fig.4. Comparing with Fig.3, the total capacitance and sensitivity are near their minimums.

4.2. Theoretical verification

Now, the reason why the $N_{\min}$ condition is valid for reducing both the total capacitance and sensitivity is discussed in the following. First, it will be verified that the $N_{\min}$ pairing condition is valid for reducing the total capacitance.

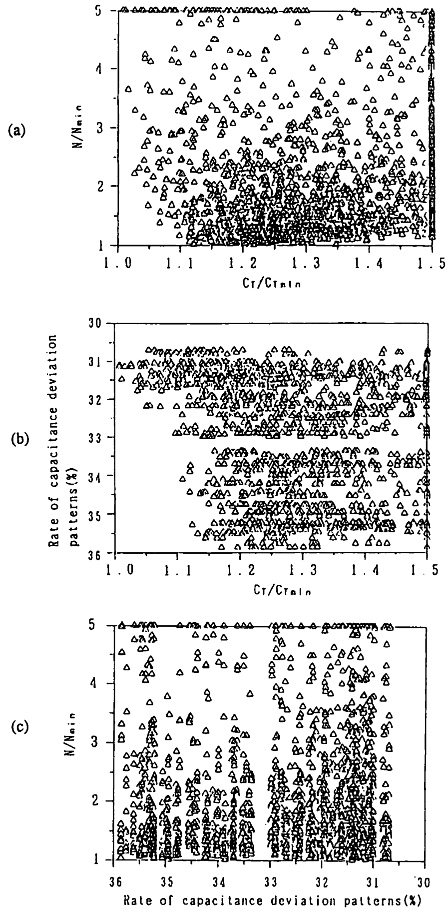


Fig.3 Distributions of three parameters without any constraints.

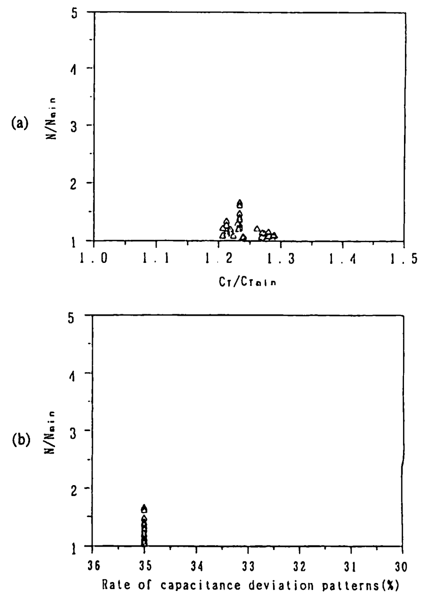


Fig.4 Distributions of three parameters with the $N_{\min}$ condition.

The total capacitance of the biquad circuit in Fig.1 is given as the sum of C_{T1} and C_{T2}. C_{T1} and C_{T2} are sum of C_{1i}(i=0~4) and C_{2i}(i=0~3), respectively. Capacitances are scaled in order to avoid overload of OpAmps. Furthermore, they are normalized so as to make the minimum capacitance to be unity. After that, C_{T1} and C_{T2} are expressed as follows [1]:

$$C_{T1} = \frac{1}{2k_1(1-cos\theta_z)} + \frac{1-cos\theta_p}{k_2(1-cos\theta_z)} + 1 \quad (1)$$

$$C_{T2} = \frac{1}{k_2} + \frac{1-r_p^2}{k_2} + \frac{1}{k_1} + 1 \quad (2a)$$

or

$$C_{T2} = \frac{1}{1-r_p^2} + 1 + \frac{k_2}{k_1(1-r_p^2)} + \frac{k_2}{1-r_p^2} \quad (2b)$$

depending on which is the minimum capacitance in C_{T2}. In Eqs.(1) and (2), k_1, k_2 are scaling factors, θ_z, r_p and θ_p are parameters of zeros and poles respectively.

The total capacitance of a high-Q section, where $r_p \approx 1$, is dominant in the total capacitance of a cascade SC filter [1]. Therefore, it is most important to reduce the total capacitance of the high-Q section. For a high-Q section, the scaling factors k_1, k_2 are approximately expressed by

$$k_1 \approx \left| \frac{(1-r_p)(1-r_p e^{-j2\theta_p})}{4(cos\theta_z - cos\theta_p)sin(\theta_p/2)} \right| \quad (3)$$

$$k_2 \approx \left| \frac{(1-r_p)(1-r_p e^{-j2\theta_p})}{2(cos\theta_z - cos\theta_p)} \right|. \quad (4)$$

Thus, k_1, k_2 increase when θ_z approaches θ_p. However, k_1 and k_2 can not be larger than 1, because r_p is close to unity [2]. Now, changes of C_{T1} and C_{T2} are examined when θz approaches θ_p.

Change of C_{T1}:

For a high-pass filter, k_1, k_2 and $(1-cos\theta_z)$ increases when θz approaches θ_p. This results in subsequent decrease in C_{T1}.

For a low-pass filter, when θz approaches θ_p, k_1 and k_2 increase, and $(1-cos\theta_z)$ decreases. The rate of change in k_1 and k_2 is given by

$$a = \left| \frac{1}{1 - \frac{d}{\cos\theta_p - cos\theta_z}} \right|. \quad (5)$$

The rate of change in $(1-cos\theta_z)$ becomes

$$b = \left| 1 - \frac{d}{1-cos\theta_z} \right| \quad (6)$$

where

$$d = (\cos\theta_z' - cos\theta_z) > 0 \quad (7)$$

and θ_z' is the zero nearest θ_p. The following inequality is held

$$|\cos\theta_p - cos\theta_z| < |1-cos\theta_z|, \; 0 < \theta_p, \theta_z < \pi. \quad (8)$$

Thus,

$$ab > 1 \quad (9)$$

and $k_1(1-cos\theta_z)$ and $k_2(1-cos\theta_z)$ increase, followed by a decrease in C_{T1}.

Change of C_{T2}:

When θ_z approaches θ_p, following an increase of k_1 and k_2, C_{T2} decreases in Eq.(2a), but increases in Eq.(2b). Since change in C_{T2}, in Eq.(2b), is much smaller than that for C_{T1} in Eq.(1), the change in C_{T1} is more dominant. As a result, C_T decreases when a high-Q pole is paired with the nearest zero. The distributions of the output noise and the total capacitance with the N_{min} pairing condition is shown in Fig.5.

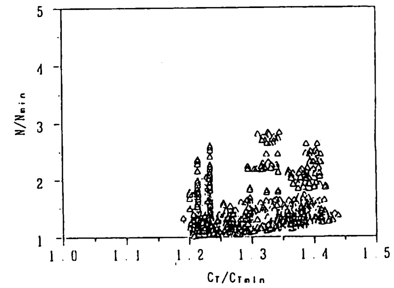


Fig.5 Distributions of the output noise and total capacitance with the N_{min} pairing condition.

Next, the reason why N_{min} ordering condition is also valid for reducing the total capacitance is discussed. An amplitude peak frequency of a biquad circuit is almost determined by a pole frequency. Therefore, the N_{min} ordering condition is realized by arranging high-Q poles and low-Q poles alternately. This means, some low-Q poles are arranged to the preceding section of high-Q poles, so that the total capacitance can be decreased.

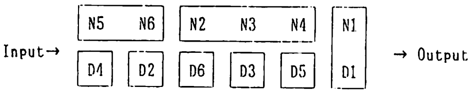
Finally, the reason why the N_{min} condition is valid for reducing sensitivity is discussed. In N_{min} pairing condition, high-Q poles are paired with zeros close to them, so that it is valid for reducing the sensitivity. The number of pairing is only one in the N_{min} condition, but there exist several pairings in the S_{min} condition, because there are no pairing constraints for low-Q poles. Therefore, although the sensitivity can be reduced by using N_{min} condition, the minimum sensitivity is not guaranteed.

5. TRADE-OFF OPTIMUM CONDITIONS

Trade-off conditions can be developed by combining the N_{min} condition with two other conditions. Depending upon applications, different trade-off optimum conditions are given by adjusting the proportion of the three conditions. Two kinds of trade-off optimum conditions are shown here.

5.1. Trade-off optimum Condition 1

In order to reduce the total capacitance, while maintaining the output noise to be at the minimum, the $C_{T_{min}}$ condition is combined with the N_{min} condition in the following way. First, poles are ordered according to the N_{min} condition, which is dominant for reducing output noise [2]. Then, zeros are assigned according to the $C_{T_{min}}$ condition. This combined condition is called the trade-off Condition 1. One example of P/O constraints satisfying Condition 1 is shown in Fig.6. The number of P/O assignments is limited to 12.



(12 assignments)

Fig.6 P/O constraints satisfying trade-off optimum Condition 1.

The distributions of three parameters under Condition 1 are shown in Fig.7, with the symbol Δ. The output noise and total capacitance are concentrated on $(1.0\sim1.8)N_{min}$ and $(1.1\sim1.25)C_{T_{min}}$. Compared with Fig.3, these results provide good trade-off between the total capacitance and output noise. A trade-off P/O assignment can be found easily from limited P/O assignments.

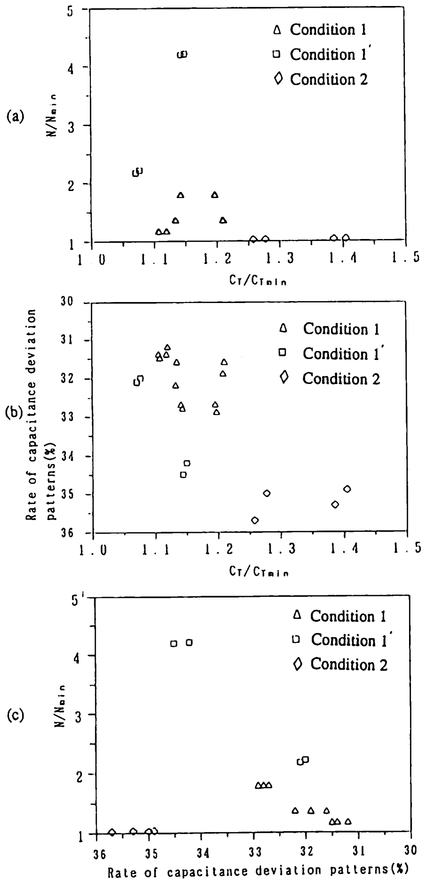
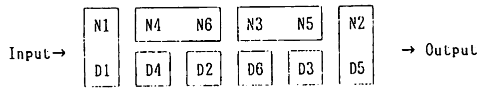


Fig.7 Distributions of three parameters with trade-off Condition 1, 1' and 2.

In order to gain more reduction in the total capacitance, a modified Condition 1, called Condition 1' is proposed by increasing the proportion of $C_{T_{min}}$ condition. One example of P/O constraints satisfying the Condition 1' is shown in Fig.8. The distributions of three parameters under Condition 1' are shown in Fig.7, with the symbol □. Total capacitances are more close to $C_{T_{min}}$, at the expense of an increase in the output noise.

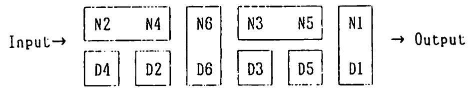


(4 assignments)

Fig.8 P/O constraints satisfying trade-off optimum Condition 1'.

5.2. Trade-off optimum Condition 2

The disadvantage of Condition 1 is that the sensitivities are far from the minimum value. To overcome this disadvantage, another condition is proposed by taking the S_{min} condition into account. That is, Condition 1 is modified by pairing high-Q poles with zeros close to them. This condition is called Condition 2. One of theOA example of P/O constraints satisfying Condition 2 is shown in Fig.9. The distributions of three parameters under Condition 2 are shown in Fig.7, with the symbol ◊ . The sensitivities are more close to S_{min} at the expense of an increase in the total capacitances. A trade-off optimum P/O assignment, witch can provide the minimums in both the output noise and sensitivity, and $1.26C_{T_{min}}$ in the total capacitance, can be easily found.



(4 assignments)

Fig.9 P/O constraints satisfying trade-off optimum Condition 2.

6. CONCLUSIONS

It has been verified that there is no P/O assignment with which three parameters can be simultaneously minimized. Trade-off optimum conditions have been proposed, under which a trade-off optimum P/O assignment can be searched out easily among the limited P/O assignments. Depending upon applications, the trade-off optimum condition can be slightly modified by adjusting the proportion of the $C_{T_{min}}$, S_{min} and N_{min}. Efficiency of the proposed trade-off optimum conditions have been confirmed through computer simulation using several kinds of filters.

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* Permanent residence: Dept. of Automatic Control Engineering, Central-South University of Technology, Changsha, China.