Convergence and Residual Error of Multi-point Active Noise Controller

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Abstract

This paper investigates the performance of a multi-point noise controller combined by individual single-output noise controllers. The convergence and residual error of multi-point noise controller are mainly investigated based on the cross-connection matrix $\mathbf{C}_{LL}(z)$ which denotes the transfer functions from the output of each adaptive filter to every noise cancelling point. The relation between the optimum solution of adaptive filters and cross-connection matrix $\mathbf{C}_{LL}(z)$ is deduced. The influence of non-diagonal components $C_{i,k}(z)(i \neq k)$ on the convergence and residual error is investigated through computer simulation. The following results are achieved. The optimum solution can be obtained as long as $\mathbf{C}_{LL}(z)$ is not singular. In the case of that $C_{i,k}(z)(i \neq k)$ is in proportion to $C_{k,k}(z)$, adaptive filters converge slowly when the ratio r $(0 \leq r < 1)$ increases. But the residual errors are almost the same. In the case of $C_{i,k}(z)(i \neq k)$ is different from proportion to $C_{k,k}(z)$, in order to obtain the same noise cancelling effect, more number of taps is needed when $C_{i,k}(z)(i \neq k)$ is far different from $C_{k,k}(z)$.

1 Introduction

A single-output noise controller has been well discussed. By using an adaptive filter and a loudspeaker, a duplicate of noise is generated which has same magnitude and opposite phase with original noise at a cancelling point. The noise can be canceled at the location of the cancelling point by interaction of duplicate noise with original noise. The adaptive filter is adapted to minimize the noise of the cancelling point.

In order to cancel noises in locations of L individual points, such as cancelling the engine noise at every site of a car, or a helicopter etc., a multi-point noise controller is needed. One simple construction of a multi-point noise controller is performed by combining individual single-output noise controllers. It is known that the independent single-output noise controllers may fight each other in an attempt to minimize residual noise at one location without regard to other locations. However, the effect and the limitation of the individual single-output noise controllers used for multi-point active noise controller are not well investigated [1-4]. In this paper, we investigate convergence and residual error of multi-point noise controller based on the cross-connection from the output of each adaptive filter to every noise cancelling point.

2 Multi-point Noise Controller

2.1 Construction and Optimal Solution of Multi-point Noise Controller

The construction of individual single-output noise controllers used for multi-point active noise controller is shown in Fig.1. Where x denotes a noise source, e_1, $\cdots$, e_L, residual errors at cancelling point 1, $\cdots$, L, y_1, $\cdots$, y_L, outputs of adaptive filters. $\mathbf{H}_L$ denotes paths from the noise source to the points where the noise needed to be cancelled, $\mathbf{W}_L$, L individual adaptive filters, $\mathbf{C}_{LL}$, cross-connection matrix from adaptive filters to cancelling points.

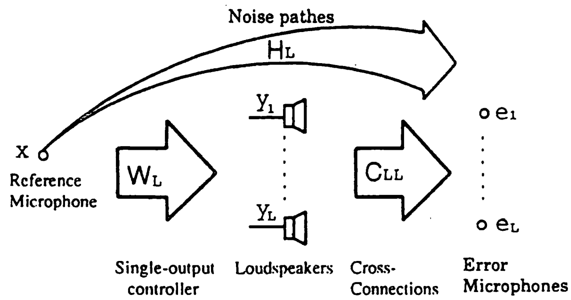

Figure 1: Constructure of multi-point noise controller

In z domain, the residual errors of n points are expressed by

$$E_i(z) = [H_i(z) - C_{i1}(z)W_1(z) - C_{i2}(z)W_2(z) - \cdots - C_{iL}(z)W_L(z)]X(z) \quad (1)$$

$$(i = 1, 2, \cdots, L)$$

When residual errors $E_1(z), \cdots, E_L(z)$ equal zero, the optimal solution of individual adaptive filters is given by

$$\begin{pmatrix} W_1(z) \\ \vdots \\ W_L(z) \end{pmatrix} = \begin{pmatrix} C_{11}(z) & \dots & C_{1L}(z) \\ \vdots & \ddots & \vdots \\ C_{L1}(z) & \dots & C_{LL}(z) \end{pmatrix}^{-1} \begin{pmatrix} H_1(z) \\ \vdots \\ H_L(z) \end{pmatrix} \quad (2)$$

Equation (2) can be briefly written as

$$\mathbf{W}_L(z) = \mathbf{C}_{LL}(z)^{-1}\mathbf{H}_L(z) \quad (3)$$

The optimal solution of individual adaptive filters exists only when $\mathbf{C}_{LL}(z)$ is not singular. We will discuss how convergence and residual errors change dependent on cross-connection matrix $\mathbf{C}_{LL}(z)$ in the following section.

3 Computer Simulation and Discussion

In order to simplify the problem but not to lose generality, we will mainly discuss a 2-point case and expand it to a L-point case in computer simulation. The model of a 2-point noise controller is shown in Fig.2. In a 2-point case, the optimal solution of individual adaptive filters is given by

$$\begin{pmatrix} W_1(z) \\ W_2(z) \end{pmatrix} = \begin{pmatrix} C_{11}(z) & C_{12}(z) \\ C_{21}(z) & C_{22}(z) \end{pmatrix}^{-1} \begin{pmatrix} H_1(z) \\ H_2(z) \end{pmatrix} = \frac{1}{D} \begin{pmatrix} C_{22}(z) & -C_{12}(z) \\ -C_{21}(z) & C_{11}(z) \end{pmatrix} \begin{pmatrix} H_1(z) \\ H_2(z) \end{pmatrix} \quad (4)$$

$$D = C_{11}(z)\, C_{22}(z) - C_{21}(z)\, C_{12}(z)$$

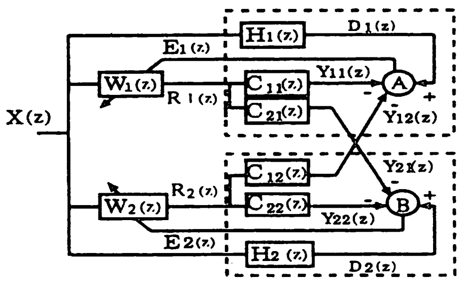

Figure 2: Model of 2-point noise controller

3.1 Adaptive Algorithm

Normalized LMS adaptive algorithm is used to update tap weight of transversal adaptive filter. The residual error at point i ($i = 1, \cdots, L$) is

$$e_i(n) = d_i(n) - \mathbf{x}^T(n)\mathbf{w}_i(n) \quad (5)$$

The tap weight of adaptive filter is updated by

$$\mathbf{w}_i(n+1) = \mathbf{w}_i(n) + \frac{\alpha}{\beta + \|\mathbf{x}(n)\|^2}\mathbf{x}(n)e_i(n) \quad (6)$$

$$\begin{aligned} \mathbf{x}(n) &= [x(n),\ x(n-1),\ \dots\ x(n-M+1)]^T \\ \mathbf{w}_i(n) &= [w_{i1}(n),\ w_{i2}(n),\ \dots\ w_{iM}(n)]^T \\ \|\mathbf{x}(n)\|^2 &= x^2(n) + x^2(n-1) + \dots \\ &+ x^2(n-M+1) \end{aligned}$$

Where, α is a step size parameter, β is a small constant and M is a number of taps. x(n) and $e_i(n)$ denote tap input vector and residual error at time n. $\|\mathbf{x}(n)\|^2$ denotes the power of the input vector. In the following simulation, noise source $x(n)$ is white noise. α=0.1,β=10^{-6} and $M = 15$ for each adapter filter.

3.2 Evaluation of noise cancellation

Effect of noise cancellation is evaluated by the average mean square errors $E_i(m)(i = 1, \cdots, L)$. $E_i(m)$ is defined by

$$E_i(m) = 10\log_{10}\left[\frac{1}{M_0}\sum_{n=(m-1)M_0+1}^{mM_0} |e_i(n)|^2\right] \quad (7)$$

$$(i = 1, 2, \quad m = 1, 2, \cdots \quad mM_0 \leq N)$$

Where, N is the total iterations. $E_i(m)$ denotes the average mean square error in every M_0 iterations. $M_0 = 20$ in the simulation.

3.3 The optimal tap weights of adaptive filters

The converged tap weights of adaptive filters $w_{ij}(n)(i = 1, 2, j = 1, \cdots M)$ are defined by the average value of $w_{ij}(n)(i = 1, 2, j = 1, \cdots M)$ in the last S iterations.

$$w_{ij} = \frac{1}{S}\sum_{n=N-S+1}^{N} w_{ij}(n) \quad (i = 1, 2 \quad j = 1, \cdots, M) \quad (8)$$

Where N denotes the total iterations. $S = 1000$ in the simulations. The theoretical optimal tap weights $\mathbf{w}_i$(i=1,2) are obtained by inverse z transfer of Eq.(2).

3.4 Transfer function of noise paths

Several different noise paths have been used in the simulations. One of them is shown in Fig. 3. $H_1(z)$ and $H_2(z)$

denote the transfer function of noise paths from the noise source to cancelling points A and B, respectively. $C_{11}(z)$ denotes the transfer function from the output of adaptive filter 1 to cancelling point A, and $C_{22}(z)$ denotes the transfer function from the output of adaptive filter 2 to cancelling point B. $C_{21}(z)$ denotes the transfer function of the output of adaptive filter 1 to the cancelling point B and $C_{12}(z)$ denotes the transfer function of the output of adaptive filter 2 to the cancelling point A.

We will discuss how the frequency characteristics of $C_{ik}(z)(i \neq k)$ affect the residual errors of cancelling points in following sections.

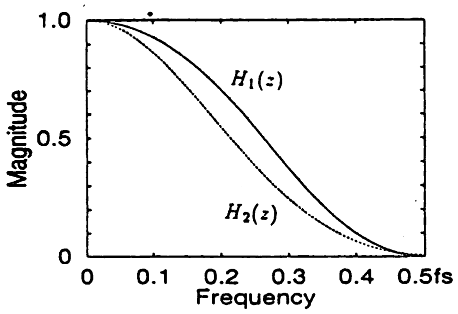

Figure 3: Magnitude-frequency response of $H_1(z)$ and $H_2(z)$

3.5 $C_{ik}(z)(i \neq k)$ is in proportion to $C_{kk}(z)$

In this case, the transfer function from the output of k-th adaptive filter to i-th ($i = 1, \cdots, i \neq k$) error microphone is in proportion to the transfer function from the output of k-th adaptive filter to k-th error microphone. The ratio r satisfies the condition of $0 \leq r < 1$. Figure 4 shows frequency characteristics of $C_{11}(z)$, $C_{22}(z)$ and the product of $C_{11}(z)$ and $C_{22}(z)$. $C_{21}(z)$ and $C_{12}(z)$ are in proportion to $C_{11}(z)$ and $C_{22}(z)$.

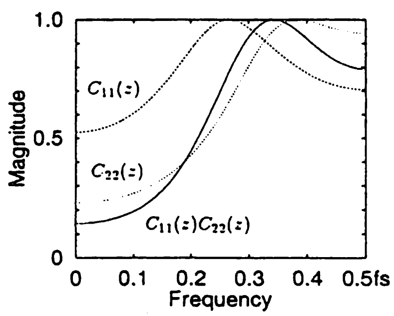

Figure 4: Frequency characteristics of $C_{11}(z)$ and $C_{22}(z)$

Figure 5 shows the mean square error when r=0, 0.5, 0.8, 0.9, respectively. First, we can see, mean square error converges slowly when r increases. This is because when $r = 0$, cross-connection matrix $C_{LL}(z)$ is a diagonal matrix, that means that there is no cross-connection among every individual adaptive filter. Each adaptive filter performs as same as in single-point case. Equation 4 corresponds to two independent individual noise controlling problems. The result is as same as one-point noise controller. When $r \neq 0$, there is cross-connection among every individual noise controller, and the individual adaptive filters may fight each other in an attempt to minimize residual noise at one location without regard to other locations. As a result, adaptive filter converges more slowly when r increases. When $r = 1$, The denominator of Eq. 4 is zero. C_{LL} becomes singular, the solution of adaptive filters do not exist. Residual errors diverged in the simulation are not shown here.

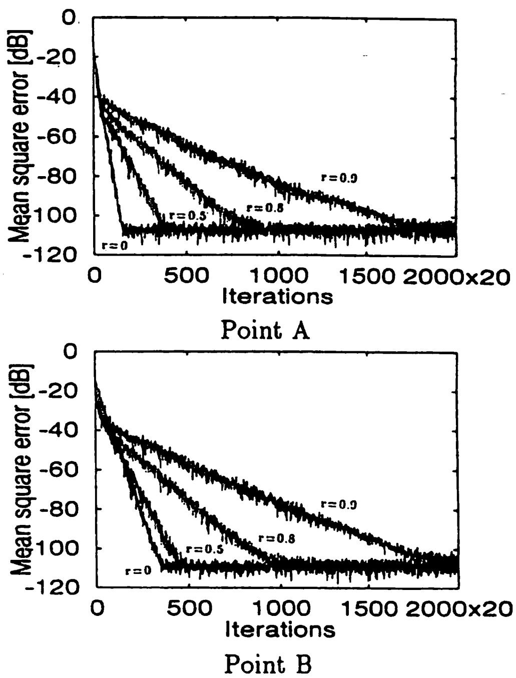

Figure 5: Meansquare errors

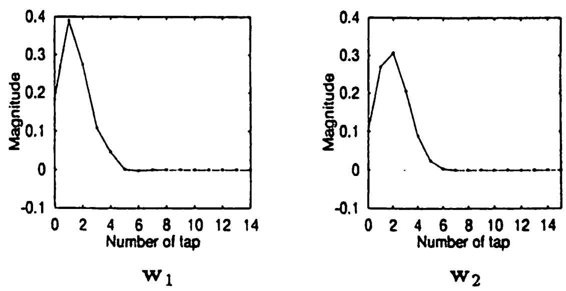

Figure 6: Tap weight vectors when r=0

From Fig. 5, we also can see that the mean square errors converge to -110dB whatever the value of r is. That means after convergence, the residual error is indepen-

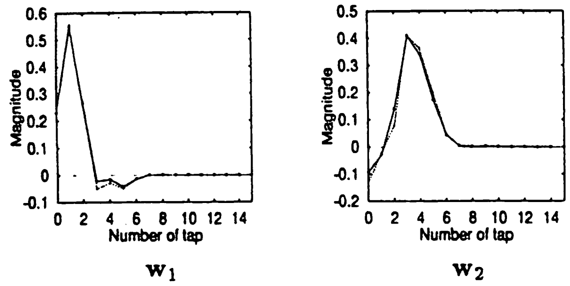

Figure 7: Tap weight vector when r=0.9

dent to cross-connection $C_{ik}, i \neq k$. Figure 6 and Fig. 7 show that the converged tap weight of adaptive filters $\mathbf{w}_1$, $\mathbf{w}_2$ when $r = 0$ and $r = 0.9$. Solid lines denote theoretical values and dot lines denote the simulation results. In fig.6, the solid line overlaps dot line. We can see 15 taps is enough for adaptive filter 1 and 2 in cases of different r, so that the residual errors can converge to same value.

3.6 $C_{ik}(z)$ $(i \neq k)$ is different from $C_{kk}(z)$

In the case of $C_{ik}(z)$ $(i \neq k)$ different from $C_{kk}(z)$, the transfer function from the output of k-th adaptive filter to i-th $(i = 1, \cdots, i \neq k)$ error microphone is different from the transfer function from the output of k-th adaptive filter to k-th error microphone. In order to simplify the discussion, the product of $C_{11}(z)$ and $C_{22}(z)$ is shown in Fig. 8. the product of $C_{21}(z)$ and $C_{12}(z)$ for four different cases a, b, c, d are shown in Fig. 8. The magnitude of a, b, c and d is half of magnitude of $C_{11}(z)C_{22}(z)$. $H_1(z)$ and $H_2(z)$ are same as in Fig. 3.

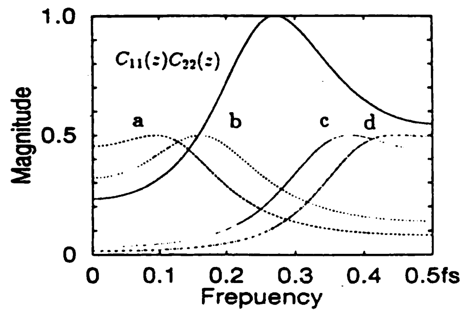

Figure 8: Frequency characteristics of $C_{21}(z)C_{12}(z)$ and $C_{11}(z)C_{22}(z)$

Figure 9 shows mean square error corresponding to the cases a, b, c, and d. We can see that residual errors increase according to the order of c, d, b, a. Mean sqare error is smaller in the case of c and d than the cases of a and b. The frequency components of $C_{21}(z)C_{12}(z)$ are all included in the frequency components of $C_{11}(z)C_{22}(z)$ in the cases of c and d, but not in the cases of a and b. Residual error increase when the frequency corresponding to the maximum magnitude of $C_{21}(z)C_{12}(z)$ are far different from that of $C_{11}(z)C_{22}(z)$.

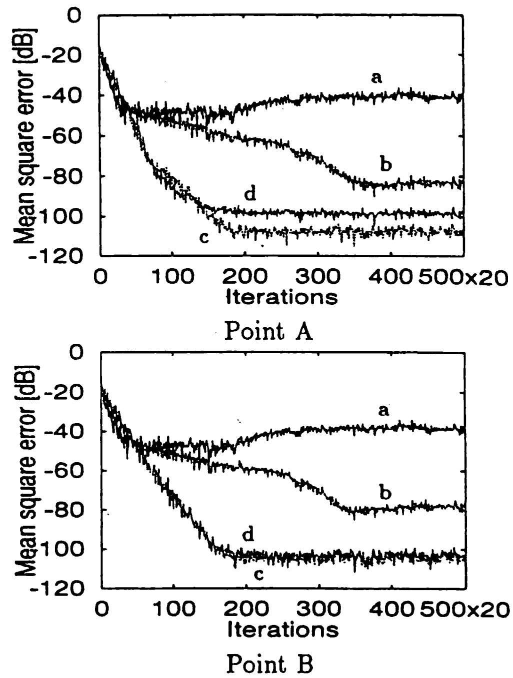

Figure 9: Mean square errors

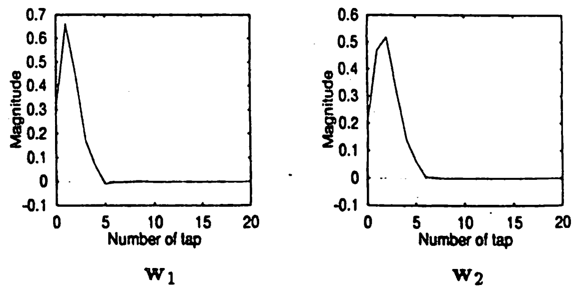

Figure 10: Theoretical tap weights in the case of c

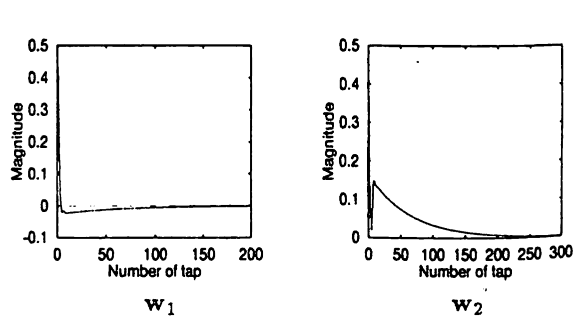

Figure 11: Theoretical tap weights in the case of a

This is because that the necessary number of taps of adaptive filter increases when the frequency corresponding to the maximum magnitude of $C_{21}(z)C_{12}(z)$ are far different from that of $C_{11}(z)C_{22}(z)$. The theoretical tap weights of adaptive filters $\mathbf{w}_1$ and $\mathbf{w}_2$ in the cases c and a are shown in Fig. 10 and Fig. 11. In the case c, the theoretical numbers of taps of $\mathbf{w}_1$ and $\mathbf{w}_2$ are less than 15, so that the residual errors of point A and B can converge to -110dB. But in the case a, the theoretical numbers of taps of $\mathbf{w}_1$ and $\mathbf{w}_2$ are more than 150 and 250 taps, respectively. $M = 15$ is not enough for both adaptive filters, so that the residual errors of point A and B converge to a large value, -40dB.

The residual errors can be further reduced by increasing the number of taps of adaptive filters in the cases a, b, c and d. Figure 12 shows the mean square error of point A and B in the case of a, when the number of taps are 150, 300 and 400, respectively. Figure 12 shows the residual errors decrease when the number of taps increase. It means that more number of taps is needed when $C_{ik}(z)$ $(i \neq k)$ is far different from $C_{kk}(z)$.

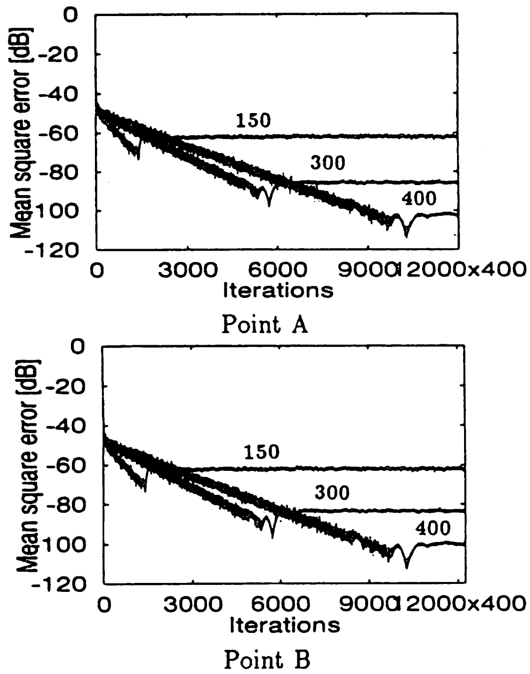

Figure 12: Mean square errors

4 Conclusion

The performance of individual single-output noise controllers used for multi-point active noise controller has been investigated in this paper. Generally speaking, the optimal solution of each individual adaptive filter can be obtained as long as cross-connection $C_{LL}(z)$ is not singular. In the case of that $C_{ik}(z)$ $(i \neq k)$ is in proportion to $C_{kk}(z)$, residual error can converge to same value as long as the number of taps is enough for different r. But adaptive filter converges slowly when r increases. in the case of that $C_{ik}(z)$ $(i \neq k)$ is different from $C_{kk}(z)$, more number of taps is needed when $C_{ik}(z)$ $(i \neq k)$ is far different from $C_{kk}(z)$ in order to get same cancelling effect.

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