Performance of An Automatic Tap Assignment Adaptive Filter for Measurement Noise and Colored Input Signal

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1 Introduction

Sub-band adaptive filters are efficient techniques for reducing computational requirements and for improving the convergence rate. Usually, the lengths of all the sub-band adaptive filters are set to be the same [1]-[5]. However, a uniform length is not always the optimum; the length of each sub-band adaptive filter may depend on the length of the impulse response of a divided unknown system in the corresponding band.

In a recent study, we proposed an Automatic-Tap-Assignment (ATA) sub-band adaptive filter configuration which enables automatic adjustment of the lengths of sub-band adaptive filters [6]. The number of taps of each sub-band adaptive filter is controlled by the mean-squared error (MSE) in each band. The number of taps increases in the bands where large errors occur and decreases in the bands where small errors occur, until the MSE in all the bands become the same. Compared with the existing uniform-length methods, the total MSE can be reduced by $5 \sim 10dB$.

In the actual application, the total number of taps is limited by hardware. When the total number of taps is fixed, the optimal tap assignment to minimize the total residual error is further investigated in this paper. The effect of the measurement noise on the optimal tap assignment is also discussed.

On the other hand, when the input signal is colored, power of input signal in each band is different. The MSE depends not only on the tap weight error but also on the input signal power. Therefore, the following problems remain. How does the input signal affect tap assignment in the ATA sub-band adaptive filter? Can the total residual error be minimized in the colored input signals? These problems are also discussed[7].

2 ATA Sub-band Adaptive Filter

2.1 Block Diagram

For simplicity, a two-band case shown in Fig.1 is taken into account in this paper. U.S. denotes an unknown system. $\mathbf{e}_0(n)$ denotes the measurement error. $\mathbf{A}_1$ and $\mathbf{A}_2$ are the analysis filters which split the full-band input signal $\mathbf{x}(n)$ and desired response $\mathbf{d}(n)$ into two-band signals. $\mathbf{x}_1(n)$, $\mathbf{x}_2(n)$, $\mathbf{d}_1(n)$, and $\mathbf{d}_2(n)$ denote the components of $\mathbf{x}(n)$ and $\mathbf{d}(n)$ in the low and high bands, respectively. In order to avoid aliasing effects, over-sampling is used. This does not lose generality in discussing efficiency of the ATA sub-band adaptive filter. $\mathbf{w}_1$ and $\mathbf{w}_2$ are the tap weights in the low and high bands, respectively. $\mathbf{y}_1(n)$ and $\mathbf{y}_2(n)$ are the outputs of the adaptive filters, and $\mathbf{e}_1(n)$ and $\mathbf{e}_2(n)$ are the residual errors. $\mathbf{S}_1$ and $\mathbf{S}_2$ are the synthesis filters which synthesize $\mathbf{y}_1(n)$ and $\mathbf{y}_2(n)$ into the full-band signal $\mathbf{y}(n)$. The difference between $\mathbf{d}(n)$ and $\mathbf{y}(n)$ is the total residual error $\mathbf{e}(n)$.

2.2 Automatic Tap Assignment

The number of taps of the two sub-band adaptive filters, denoted by $N_1(n)$ and $N_2(n)$, are initially set to be the same

$$N_1(0) = N_2(0) \tag{1}$$

The mean-squared error (MSE) of the k-th adaptive filter is given by

$$E_k(n) = \frac{1}{M_0} \sum_{m=n-M_0+1}^{n} e_k^2(m), \quad k = 1,2 \tag{2}$$

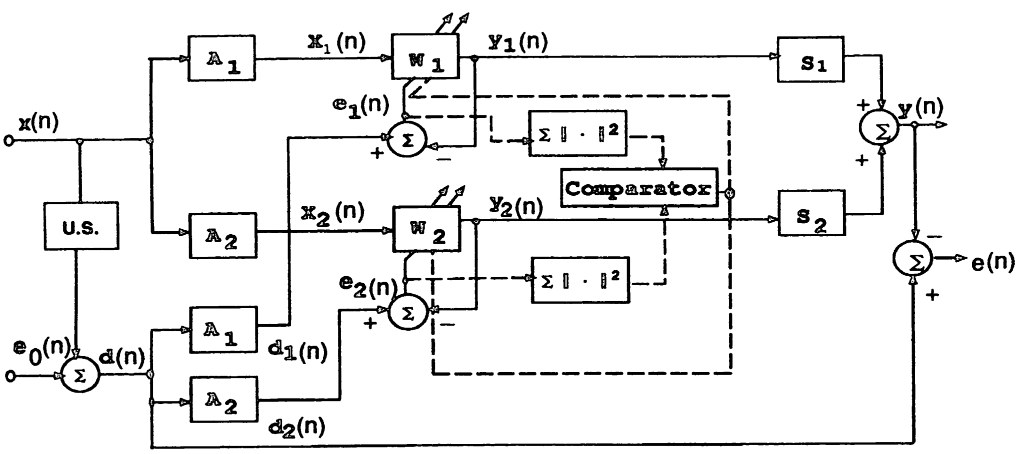

Figure 1: ATA sub-band adaptive filter configuration in two-band case.

$E_k(n)$ is calculated and compared at every M_0 samples. The output of the comparator controls the tap assignment. When the MSE of band 1 is larger than that of band 2, The number of taps is controlled by the following equations.

$$\begin{aligned} If\ \ E_1(n) &> E_2(n)(1+\delta),\ \ n = iM_0 \\ then\ \ N_1(n+1) &= N_1(n)+1, \\ N_2(n+1) &= N_2(n)-1 \end{aligned} \tag{3}$$

where, δ is a small positive constant to avoid the viburation in tap assignment. The initial weights at the next iteration are determined by

$$\begin{aligned} \mathbf{w}_1(n+1) = \mathbf{w}_1(n), 0 \le k < N_1(n+1) \\ \mathbf{w}_1(n+1) = 0, k = N_1(n+1) \\ \mathbf{w}_2(n+1) = \mathbf{w}_2(n), 0 \le k \le N_2(n+1), \end{aligned} \tag{4}$$

When the MSE of band 2 is larger than that of band 1, The tap assignment method is the same as mentioned above. The tap assignment will continue until the following condition is satisfied.

$$|E_1(n) - E_2(n)| \le \delta \tag{5}$$

3 Optimal Tap Assignment

3.1 Relation Between Total Residual Error and Tap Assignment

The relation between the total residual error and tap assignment is investigated in the cases of zero and non-zero measurement error. The residual error of the k-band $\mathbf{e}_k(n)$ is given by

$$\begin{aligned} \mathbf{e}_k(n) &= \mathbf{d}_k(n) - \mathbf{w}_k^H(n)\mathbf{X}_k(n) \\ &= [\mathbf{w}_{k0}^H(n) - \mathbf{w}_k^H(n)]\mathbf{X}_k(n) \end{aligned} \tag{6}$$

where $\mathbf{w}_{k0}^H(n)$ denotes the ideal tap weights of the k-band adaptive filter, $\mathbf{X}_k(n)$ is the input vector. When the input signal is white, the power of the input signal is almost the same in each band, and the residual error mainly depend on the tap weight error. The MSE is small in the band which has enough taps, but is large in the band which is shortage in taps. By increasing taps in the band which has small MSE, and decreasing taps in the band which has large MSE, the total residual error can be minimized. Therefore, the optimal tap assignment is obtained. We will use an example to verify this conclusion.

3.2 Unknown System

An unknown system is a 10th-order all pole IIR filter. Its transfer function is expressed as

$$H(z) = K \prod_{p=1}^{5} \frac{1}{1 - 2r_p cos\theta_p z^{-1} + r_p^2 z^{-2}} \tag{7}$$

where K is a scaling factor. The amplitude and impulse response of the system are shown in Fig.2.

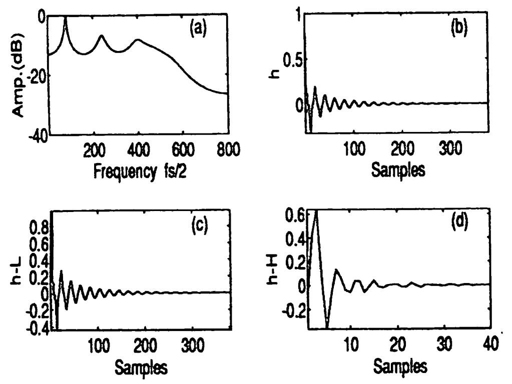

Figure 2: Amplitude and impulse response of the unknown system. (a) Amplitude-frequency response. (b) Impulse response in the full-band (h). (c) Impulse response in the low-band (h-L). (d) Impulse response in the high-band (h-H).

3.3 Simulation Conditions

In the simulation, polyphase structures were used in the analysis and synthesis filter banks [8]. A 41-tap quadrature mirror filter (QMF) was used as a prototype filter for the two-band case. Reconstruction error of analysis/synthesis system is less than $\pm 0.11dB$. The normalized LMS algorithm [9] was used in adaptation.

$$e_k(n) = d_k(n) - \mathbf{w}_k^H(n)\mathbf{X}_k(n), \qquad (8)$$

$$\mathbf{w}_k(n+1) = \mathbf{w}_k(n) + \frac{\alpha}{\varepsilon + |\mathbf{X}_k(n)|^2}\mathbf{X}_k(n)e_k^H(n) \qquad (9)$$

where $[\cdot]^H$ denotes the Hermitian transposition, $\alpha = 0.05$, and $\varepsilon = 10^{-10}$. The input signal $\mathbf{x}(n)$ is a white noise with zero mean and variance $\sigma^2 = 0.085(-10.76dB)$.

3.4 Simulation Result of ATA Sub-band Adaptive Filter

In the ATA sub-band adaptive filter, the number of taps in two sub-band adaptive filters are initially set to the same, $N_1(0) = N_2(0) = 50$. During the adaptation process, $N_1(n)$ and $N_2(n)$ are adjusted according to Eq.(3) with $M_0 = 10$. After convergence, the number of taps in two adaptive filters are $N_1(E) = 93$ and $N_2(E) = 7$, in the case of non-measurement noise. The tap weight vectors $\mathbf{w}_1$ and $\mathbf{w}_2$, and the number of taps $N_1(E)$ and $N_2(E)$ are shown in Fig. 3.

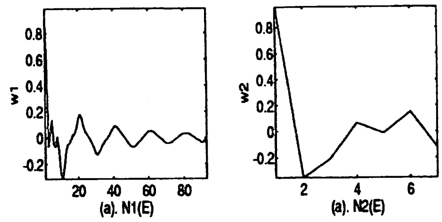

Figure 3: (a). Tap weights of low-band adaptive filter. (b). Tap weights of high-band adaptive filter.

3.5 Optimal Tap Assignment in Zero Measurement Error Case

The relation between the total residual error and the lengths of the sub-band adaptive filters is investigated. For the case of zero measurement error, the simulation result is shown in Fig.4.

The abscissa denotes the number of taps in the low-band adaptive filter. The ordinate denotes the total MSE. In the simulation, the number of total taps was set to 100. Thus the number of taps in the high-band adaptive filter is $N_2(n) = 100 - N_1(n)$. For getting an average value of the total MSE, the MSEs of final 1000 samples of $e(n)$ are used. the total MSE is $-22.08dB$, at $N_1(n) = 50$, which is the uniform-length method case. The total MSE decreases when $N_1(n)$ increases. At $N_1(n) = 93$, the total MSE attains the minimum value, $-27.96dB$. With further increase in $N_1(n)$, the total MSE also increases.

This result is consistent with the result of the ATA sub-band adaptive filters. It is confirmed that the optimal tap assignment can be obtained by the ATA sub-band adaptive filter to minimize the total MSE. Compared with the uniform length sub-band adaptive filter, the total MSE reduced by about 5dB in the ATA sub-band adaptive filter.

3.6 Optimal Tap Assignment in Non-zero Measurement Error Case

The relation between total residual error and tap assignment, when measurement error $e_0(n)$ is not zero, is shown in Fig.5. $e_0(n)$ is a white noise with

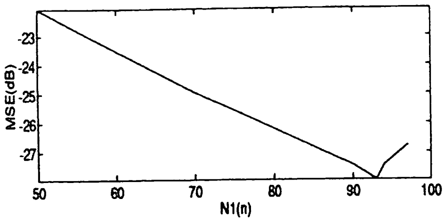

Figure 4: Relation between the total MSE and the number of taps in the case of zero measurement error

zero mean and variance σ^2 = .0085($-20.76dB$), but no correlation with input signal $\mathbf{x}(n)$. At $N_1(n)$ = 88, the total MSE achieves the minimum value, $-20.60dB$. In the same simulation conditions, the tap assignment of ATA sub-band adaptive filter after convergence is: $N_1(n)$ = 88, $N_1(n)$ = 12. This means the optimal tap assignment can also be obtained when the measurement error occurs.

The optimal tap assignment is slightly changed when measurement error occurs. The preciseness of the system identification decreases due to the measurement error, but the total residual error can also be minimized.

Compared with the uniform length sub-band adaptive filter, the total MSE reduced by about 1.5dB in the ATA sub-band adaptive filter. It seems not so efficient to reduce the total MSE by tap assignment as that in non-measurement case.

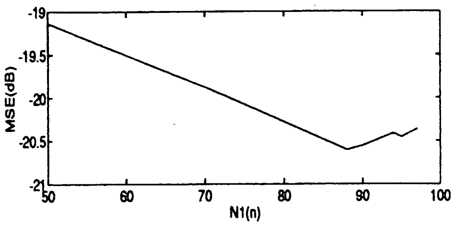

Figure 5: Relation between the total MSE and the number of taps when the measurement error is included.

4 Total Residual Error for Colored Input Signal

4.1 Effects of Colored Input Signal on Tap Assignment

Effects of colored input signal on tap assignment is investigated in this section. As shown in Eq.(6), the residual error of each sub-band adaptive filter depends on the tap weight error and the input signal. When the input signal is colored, the power of the input signal is different in each band, therefore, the residual error of each band deponds not only on the tap weight vector error, but aslo on the input signal. Because the tap assignment is controlled by the MSE in the ATA adaptive filter, the colored input signal will affect tap assignment.

In the ATA adaptive filter, the number of taps and tap weights are adjusted to minimize the total residual error. In the colored input signal case, tap assignment depends not only on the impulse response length of an unknown system in each band, but also on the power of the input signal in the corresponding band. Therefore, the total residual error is minimized, even though the unknown system is not exactly identified.

4.2 Colored Input Signal

A speech signal shown in Fig.6 is used as the input signal to investigate the total residual error and system identification error of the ATA adaptive filter. The variance of the speech signal is σ^2 = .085($-10.76dB$). The signal power concentrates in the low frequency band.

4.3 Total Residual Error in Colored Input Signal Case

The total residual error of the ATA sub-band adaptive filter is investigated in the colored signal case, using the unknown system and simulation conditions shown in section 3.2 and 3.3.

In order to avoid the effect of slow convergence rate in colored input, we use the tap weights shown in Fig.3 as the initial tap weight values. This not loss the correctness of our discussion.

the ATA sub-band adaptive filter is further adapted for the speech signal shown in Fig.6. The weight vectors of $\mathbf{w}_1$ and $\mathbf{w}_2$, the number of taps $N_1(E)$ and $N_2(E)$ after 4000 iterations are shown

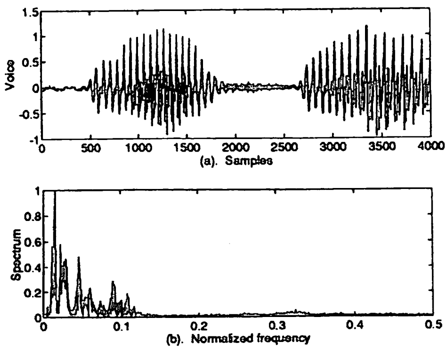

Figure 6: (a). Speech signal (b). Spectrum of the speech signal

in Fig.7 (a) and (b). The total residual error $e(n)$ is shown in Fig.7 (c). The number of taps in the low-band $N_1(E)$ has increased by 4 to 97, due to the speech signal power concentrated on the low-band. Because the input signal power is large in voice period, $e(n)$ is large in voice period compared with that in non-voice period.

In order to evaluate preciseness of the system identification of the ATA adaptive filter, The system identification error is calculated. in Fig. 1, letting $e_0(n) = 0$, $x(n)$ is an impulse signal, $\mathbf{w}_1$ and $\mathbf{w}_2$ are fixed on the value after convergence, the total residual error $e(n)$ denotes the impulse response error. The mean squared impulse response error is defined as the system identification error.

The system identification error caused by the initial tap value is $-7.24dB$. This error caused by the tap weights shown in Fig.7 is $-6.29dB$. Thus, the preciseness of the system identification is degraded when the input signal is colored.

4.4 Total Residual Error in White Noise Input Case

In order to compare the performance of the ATA sub-band adaptive filters trained using the speech signal and the white noise, the following simulation is carried out. Using the tap weight shown in Fig.3 as the initial tap weight values, the ATA sub-band adaptive filter is further adapted for the white noise input. The value of $\mathbf{w}_1$, $\mathbf{w}_2$, $N_1(E)$ and $N_2(E)$ after 4000 iterations are shown in Fig.8 (a) and (b). In Fig.2, $\mathbf{w}_1$, $\mathbf{w}_2$ are fixed to these values, letting $x(n)$ is the speech signal shown in Fig.6, the total residual error for the speech signal is calculated and shown in Fig.8 (c). Comparing with the total residual error shown in Fig.7 (c), we can find that $e(n)$ of voice period is smaller when the ATA sub-band adaptive filter is adapted by the speech signal. It is confirmed the ATA sub-band adaptive filter can minimize the total residual error in the colored input case.

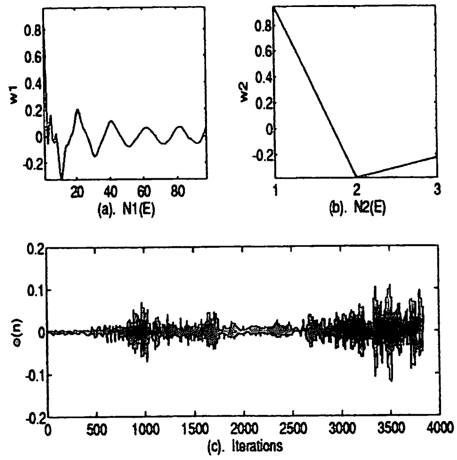

Figure 7: (a). Tap weights of low-band adaptive filter. (b). Tap weights of high-band adaptive filter. (c). Total residual error.

The system identification error caused by the tap weights shown in Fig.8 is $-7.46dB$. Comparing with the initial system identification error $-6.29dB$, the preciseness of the system identification are further improved.

The results show that, in the colored input signal case, the total residual error can minimized even though the system is not exactly identified. For the application of the noise cancellation and echo cancellation, the purpose is to minimize the total residual error. Therefore, the ATA sub-band adaptive filter is efficient for these applications.

4.5 Tracking Capability in Colored Input Signal Case

The tap assignment to minimize the total residual error changes depending on the input signal

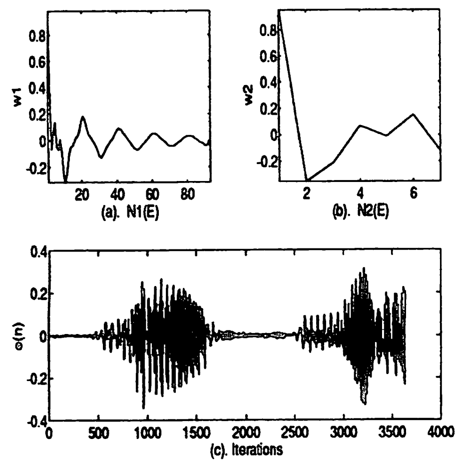

Figure 8: (a). Tap weights of low-band adaptive filter. (b). Tap weights of high-band adaptive filter. (c). Total residual error for speech signal.

spectrum. The tracking capability of the ATA sub-band adaptive filter when the spectrum of input signal changes is discussed here.

The optimal tap assignment is dependent on the divided unknown system when the input signal is white noise. When the input signal is colored, the optimal tap assignment is slightly modified in order to minimize the residual error. In our simulation, 10 iterations are needed to change one tap. If 20 taps are reassigned due to the change of the input signal spectrum, it needs 200 iterations to track this change. When sampling frequency is 8kHz, it requires 1/40 second.

5 Conclusions

The performance of of the ATA sub-band adaptive filter is further investigated for measurement noise and the colored inputs signals. For the white inputs, it is confirmed that when the total number of taps is fixed, the ATA sub-band adaptive filter can find the optimal taps assignment to minimize the total residual error. The same result is also obtained when the measurement error is added to the output of the unknown system. For the colored input signal, the total residual error can minimized, even though the unknown system is not exactly identified. It shows that the ATA sub-band adaptive filter is efficient in noise cancellation and echo cancellation.

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