Automatic Adjustment of the Lengths of Sub-band Adaptive Filters

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Abstract This paper proposes a method which enables automatic adjustment of the length of sub-band adaptive filters. In this method, the number of taps of the adaptive filter in each band is controlled by the mean-squared error. The number of taps increases in the bands which have large errors, but decreases in the bands which have small errors, until the residual errors in all the bands become the same. In this way, the length of each sub-band adaptive filter is roughly proportional to the length of the impulse response of the unknown system in the corresponding band. The effectiveness of the proposed method has been confirmed through computer simulation. Compared with the existing uniform-length and full-band methods, the convergence rate is improved and the mean-squared error is 5 $\sim$ $10dB$ smaller in the proposed method. The tracking capability of proposed method for a time-varying unknown system is almost the same as that of the uniform-length and full-band methods.

1 Introduction

Sub-band adaptive filters are efficient techniques to reduce computational requirements and to improve the convergence rate. So far, the lengths of all the sub-band adaptive filters have been set to be the same [1]-[5]. However, a uniform length is not always the optimum; the length of each sub-band adaptive filter may depend on the length of the impulse response of a divided unknown system in the corresponding band. The question arises on how to adjust the lengths of sub-band adaptive filters according to the characteristics of an unknown system.

In this paper, we present a method which enables automatic adjustment of the lengths of sub-band adaptive filters. The basic structure of the proposed method is described in Section 2. The analysis and synthesis filter banks and adaptation algorithm are introduced in Section 3. The effectiveness of the proposed method is confirmed through computer simulation. The simulation results are shown and discussed in Section 4. Finally, in Section 5 we come to the following conclusions. Compared with the uniform-length and full-band methods, the convergence rate is improved and the mean-squared error (MSE) is 5 $\sim$ $10dB$ smaller in the proposed method. The tracking capability of the proposed method for a time-varying unknown system is almost the same as that of the uniform-length and full-band methods.

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2 Automatic Adjustment of Filter Lengths

To demonstrate how the proposed method works, we discuss the two-band case as shown in Fig.1. In Fig.1, U.S. denotes the unknown system. $\mathbf{A}_1$ and $\mathbf{A}_2$ are analysis filters which split the full-band input signal $\mathrm{x}(n)$ and desired response $\mathrm{d}(n)$ into two-band signals. $\mathrm{x}_1(n)$, $\mathrm{x}_2(n)$, $\mathrm{d}_1(n)$, and $\mathrm{d}_2(n)$ denote the components of $\mathrm{x}(n)$ and $\mathrm{d}(n)$ in the low and high bands, respectively. In order to avoid the influence of the aliasing components, over-sampling is used. $\mathbf{W}_1$ and $\mathbf{W}_2$ are the tap weights in the low and high bands, $\mathrm{y}_1(n)$ and $\mathrm{y}_2(n)$ the outputs of the adaptive filters, and $\mathrm{e}_1(n)$ and $\mathrm{e}_2(n)$ the residual errors. $\mathbf{S}_1$ and $\mathbf{S}_2$ are synthesis filters which synthesize $\mathrm{y}_1(n)$ and $\mathrm{y}_2(n)$ into the full-band signal $\mathrm{y}(n)$. The difference between $\mathrm{d}(n)$ and $\mathrm{y}(n)$ is $\mathrm{e}(n)$, the residual error in the full-band.

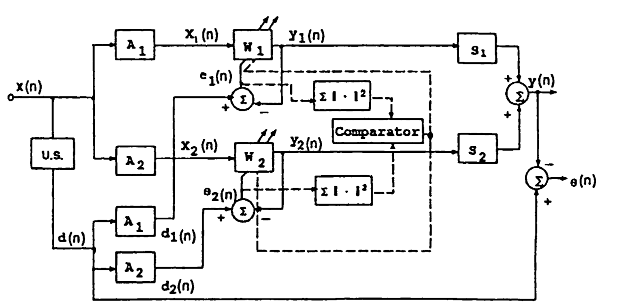

Figure 1: Automatic adjustment of filter length in the two-band case

The lengths of the two sub-band adaptive filters $N_1(n)$ and $N_2(n)$ are initially set to the same

$$N_1(0) = N_2(0) \tag{1}$$

The MSE of the k-th adaptive filter is

$$E_k(n) = \frac{1}{M_0} \sum_{m=n-M_0+1}^{n} e_k^2(m), \quad k = 1,2 \tag{2}$$

$E_k(n)$ are calculated and compared at every M_0 samples. The output of the comparator controls the filter length adjustment. The filter length increases in the band which has larger MSE, but decreases in the other. The number of taps is controlled one by one.

$$\begin{aligned} If \;\; E_1(n) > E_2(n), \quad n = iM_0, \quad i = 1, \;,\ldots \\ then \;\; N_1(n+1) = N_1(n) + 1, \\ N_2(n+1) = N_2(n) - 1 \end{aligned} \tag{3}$$

The initial weights at the next iteration become

$$\begin{aligned} w_{1k}(n) = w_{1k}(n), \quad 0 \le k \le N_1(n) \\ w_{1k}(n) = 0, \quad k = N_1(n) + 1 \\ w_{2k}(n) = w_{2k}(n), \quad 0 \le k \le N_2(n+1) \end{aligned} \tag{4}$$

The length adjustment will continue until $E_1(n)$ and $E_2(n)$ become the same.

3 Analysis and Synthesis Filter Banks and Adaptation algorithm

3.1 Analysis and Synthesis Filter Banks

Polyphase structures were used in the analysis and synthesis filter banks [6]. In the simulation, we used a 41-tap quadrature mirror filter (QMF) as a prototype filter for the two-band case. The amplitude-frequency response of the prototype filter $|H(e^{j\omega})|$ is shown in Fig.2 (a). The reconstruction error of this analysis/synthesis system is less than ± 0.11 dB as shown in Fig. 2 (b). The amplitude-frequency responses of A_1 and A_2 are $|H(e^{j\omega})|$ and $|H(e^{j\omega+\pi})|$, respectively. The amplitude-frequency responses of S_1 and S_2 are the same as A_1 and A_2, respectively.

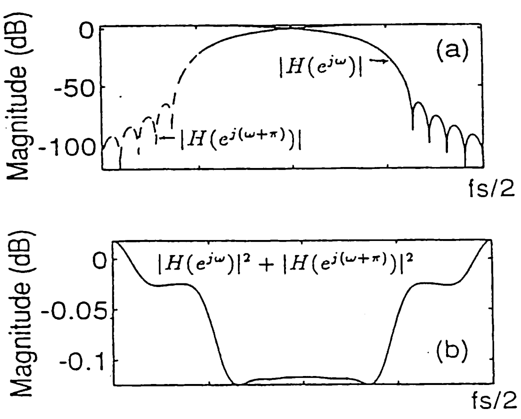

Figure 2: (a) Amplitude-frequency response of prototype filter, (b) Analysis/synthesis reconstruction error.

Table 1 Poles of the unknown system.

p	r_p	θ_p (π)
1	0.99	0.1
2	0.95	0.3
3	0.9	0.5
4	0.8	0.6
5	0.8	0.7

3.2 Adaptation Algorithm

The normalized LMS algorithm [7] was used in our simulation.

$$e_k(n) = d_k(n) - \mathbf{W}_k^H(n)\mathbf{x}_k(n), \quad k = 1, 2 \tag{5}$$

$$\mathbf{W}_k(n+1) = \mathbf{W}_k(n) + \frac{\alpha}{\varepsilon + |\mathbf{x}_k(n)|^2}\mathbf{x}_k(n)e_k{}^H(n) \tag{6}$$

where $[\cdot]^H$ denotes the Hermitian transposition, $\alpha = 0.05$ and $\varepsilon = 10^{-10}$.

4 Simulation Results

Several unknown systems were used in the simulation to confirm the effectiveness of the automatic length adjustment. Two examples are shown in the following to demonstrate the residual error, convergence rate and tracking capability of the proposed method. The input signal $\mathbf{x}(n)$ is a white noise in following simulations.

4.1 Unknown System

A 10th-order all poles IIR filter was used as the unknown system. Its transfer function can be expressed as

$$H(z) = K \sum_{p=1}^{5} \frac{1}{1 - 2r_p cos\theta_p z^{-1} + r_p^2 z^{-2}} \tag{7}$$

where K is a scaling factor. The values of r_p and θ_p are shown in Table 1.

The amplitude and impulse response of the unknown system are shown in Fig.3. h-L and h-H denote the impulse responses in the low and high bands, respectively. When the impulse response approximates 0, the number of samples is defined as the length of the impulse response. Fig. 3. shows that the ratio of the lengths of the impulse responses h-L/h-H is in the order of 10.

4.2 Residual Error and Convergence Rate

In the proposed method the lengths of adaptive filters are initially set to

$$N_1(0) = N_2(0) = 50 \tag{8}$$

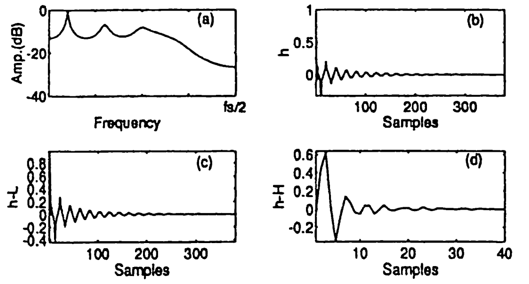

Figure 3: Amplitude and impulse response of the unknown system. (a) Amplitude-frequency response. (b) Impulse response in the full-band (h). (c) Impulse response in the low-band (h-L). (d) Impulse response in the high-band (h-H).

During the adaptive process, $N_1(n)$ and $N_2(n)$ are adjusted according to Eq.(3), and $M_0 = 10$. After convergence, the lengths of the two adaptive filters are

$$N_1(n) = 93, \quad N_2(n) = 7 \tag{9}$$

The simulation results are shown in Fig. 4.

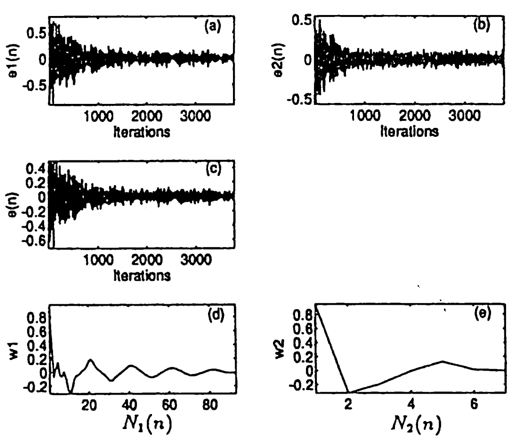

Figure 4: Residual errors and tap weights in the proposed method. (a) Residual error of the low-band adaptive filter ($e_1(n)$). (b) Residual error of the high-band adaptive filter ($e_2(n)$). (c) Total residual error (e(n)). (d) Tap weights of the low-band adaptive filter ($\mathbf{W}_1$). (e) Tap weights of the high-band adaptive filter ($\mathbf{W}_2$).

The residual errors of the two adaptive filters become almost the same after convergence. The ratio of the filter lengths $N_1(n)/N_2(n)$ is in the order of 10, consistent with the ratio of lengths of the impulse responses h-L/h-H.

The simulation results of the uniform-length method, with 50 taps in each adaptive filter, are shown in Fig.5. $e_1(n)$ and $e(n)$ are larger than that in the proposed method.

In the proposed method, the lengths of all the sub-band adaptive filters can be adjusted to consist with the lengths of the impulse responses of the divided unknown system in these bands. Thus, when the lengths of the impulse response

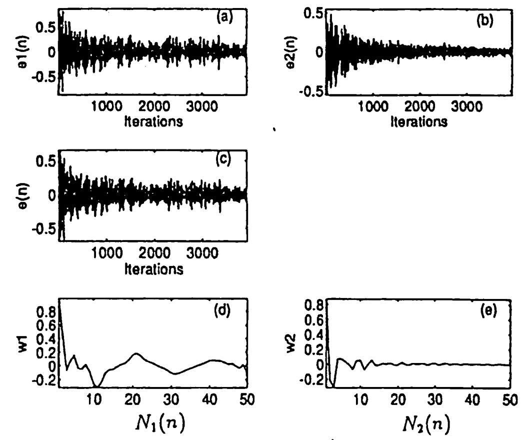

Figure 5: Residual errors and tap weights in the uniform filter lengths. (a) Residual error of the low-band adaptive filter ($e_1(n)$). (b) Residual error of the high-band adaptive filter ($e_2(n)$). (c) Total residual error (e(n)). (d) Tap weights of the low-band adaptive filter ($\mathbf{W}_1$). (e) Tap weights of the high-band adaptive filter ($\mathbf{W}_2$).

in all the bands are unequal, the residual error can be reduced by using the proposed method.

The residual error and tap weights of the full-band adaptive filter, with 100 taps and $\alpha = 0.05$, are shown in Fig. 6. This result is inferior to that obtained by the proposed method.

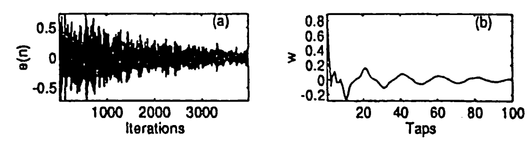

Figure 6: Residual error and tap weights in full-band adaptive filter. (a) Residual error ($e(n)$). (b) Tap weights ($\mathbf{W}$).

The MSE of the proposed, uniform-length and full-band method are shown in Fig. 7. The results show that the proposed method is superior to the others. After convergence, the MSE of proposed method is $5 \sim 10dB$ smaller than that of the others.

We have also done simulations with the unknown systems which have longer impulse response in the high band, or have the same lengths of the impulse responses in all the bands. The results show that, when the lengths of the impulse responses in all bands are unequal, the proposed method is evidently superior to the uniform-length method. When the lengths of the impulse responses in all the bands are equal, the same results are obtained.

4.3 Tracking Capability

A 2nd-order IIR filter whose pole position changes from $\pi/4$ to $3\pi/4$ at $n = n_j$ was used as the unknown system to

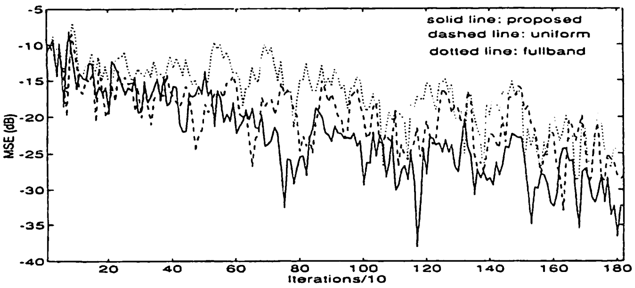

Figure 7: Mean-squared errors of the proposed, uniform-length and full-band methods.

examine the tracking capability. The amplitude response of the unknown system is shown in Fig. 8. The MSE of the propose method, the uniform-length and full-band method are shown in Fig. 9. In the simulation, the length of adaptive filter is 30 taps in full-band method, 15 taps in the uniform-length method. In the proposed method, the lengths of the adaptive filters are initially set to

$$N_1(0) = N_2(0) = 15 \tag{10}$$

After convergence, the lengths of adaptive filters are

$$N_1(n) = 25, \quad N_2(n) = 5, \qquad n < n_j \tag{11}$$

Here the lower limit of the number of taps is 5 in the simulation. When $n = n_j$, the unknown system changes. The lengths of the sub-band adaptive filters change again to follow the unknown system. After convergence again, the lengths of the adaptive filters are

$$N_1(n) = 5, \quad N_2(n) = 25, \qquad n > n_j \tag{12}$$

The MSE of the proposed method is $5 \sim 10dB$ smaller than that of the uniform-length method. The tracking capabilities of the proposed method and the uniform-length method are almost the same.

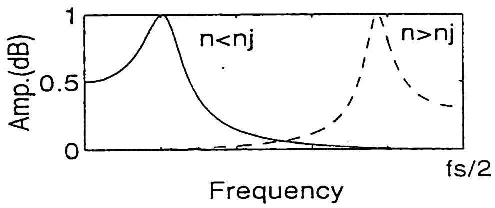

Figure 8: Amplitude response of the unknown system with a jumping.

5 Conclusions

An automatic filter-length-adjustment method has been proposed. The lengths of all the sub-band adaptive filters are controlled by the MSE. After convergence, the length of each sub-band adaptive filter is roughly proportional to the length

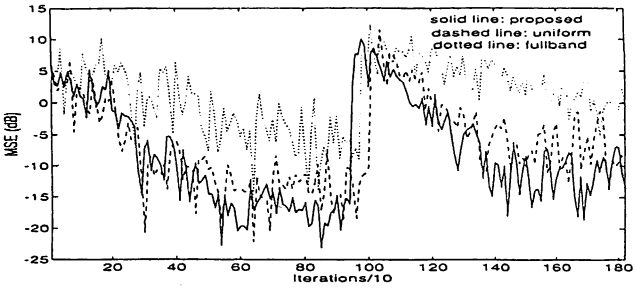

Figure 9: Tracking capabilities with the proposed, uniform-length and full-band methods.

of the impulse response of the divided unknown system in the corresponding band. By using the proposed method, the MSE is reduced by 5~10dB and the same tracking capability is achieved, in comparison with the uniform-length method.

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