A Polyphase and FFT Realization of Modulation Sub-band Adaptive Filter with Minimum Sampling Rate
変調形サブバンド適応フィルタのポリフェイズ&ＦＦＴ構成

H.Sakaguchi　K.Nakayama　A.Hirano
坂口 宏明　中山 謙二　平野 晃宏

Dept. of Electrical and Computer Eng., Faculty of Eng., Kanazawa Univ.
nakayama@t.kanazawa-u.ac.jp

ABSTRACT

A sub-band adaptive filter, in which modulation and demodulation are employed, were proposed [1],[2]. The sampling rate can be minimized, while no aliasing occurs. This means that the sampling rate can be set to just twice of the channel bandwidth. In this paper, a polyphase and FFT realization of this sub-band adaptive filter is proposed. Main difference between the polyphase and FFT realization for the transmultiplexer [3] and the proposed method is the following. In the former case, the sampling rate reduction is the same as the number of the sub-bands. In the latter case, however, they are different, so the conventional polyphase and FFT realization cannot be applied. In this paper, a new realization for the modulation sub-band adaptive filter is proposed.

あらまし

変復調を用いるサブバンド適応フィルタが提案されている。標本化周波数は折り返しなく最小化できる。即ち、各帯域幅の２倍に設定できる。本論文では、このサブバンド適応フィルタのポリフェイズ&ＦＦＴ構成法を提案する。サブバンド数Mと標本化周波数のダウンサンプル率Kが異なるため、ポリフェイズフィルタがバンド毎に異なってしまう。このため、従来のトランスマルチプレクサに用いられているポリフェイズ&ＦＦＴ法は適用できない。このポリフェイズフィルタをトランスバーサルフィルタ部とＦＦＴ部に分けることにより、各バンドでポリフェイズフィルタを共用し、計算量を減らす方法を提案している。

1. Introduction

Recently, multimedia technology has been developed, which make communication using both audio and image easy. In this case, if loud speakers are used, acoustic echo causes some problem. Audio echo cancelers are used for this reason. However, the impulse response length of room acoustic characteristics is very long. Therefore, a very high order adaptive filters are required, and at the same time, sub-band adaptive filters become very important. Computational complexity and convergence speed are sufficiently improved [4],[5].

One problem in the sub-band adaptive filter is how to determine the sampling rate. Since, the aliasing causes some distortion, the sampling rate must be higher than twice of the bandwidth of each sub-band. At the same time, computational complexity must be minimized. The modulation sub-band adaptive filter can satisfy these requirements [1],[2]. The sampling rate can be set to just the twice of the bandwidth of each sub-band. Let the number of sub-bands be M, and the sampling rate reduction be $1/K$, then M/K can be selected as any rational number. Additional requirement of computation is very small.

On the other hand, the Polyphase and FFT realization of a filter-bank, which was applied to the

TDM-FDM transmultiplexers [6], and the sub-band coding [7], is very efficient in order to reduce computational complexity of the filter bank. The filter bank can be realized with the lower sampling rate. However, in this case, the number of sub-bands M and the sampling rate reduction $1/K$ should be the same, that is $M = K$. On the other hand, in the modulation sub-band filter, M is not equal to K, rather $M > K$, and M/K is a rational number. Thus, the conventional polyphase and FFT realization cannot be applied to the modulation sub-band adaptive filter.

In this paper, a new method for polyphase and FFT realization is proposed.

2. Modulation Sub-band Adaptive Filter

Figure 1 shows a block diagram of the modulation sub-band adaptive filter [2]. The analysis filter bank consists of complex filters $F_i(z)$, the synthesis filter bank consists of complex filters $G_i(z)$. The number of sub-band is M and the sampling rate reduction is $1/K$. The carrier signals applied to both modulator (MOD) and demodulator (DEM) are complex. The input signal $x(n)$ is real. In the analysis filter bank and the modulator, the signals are represented with complex number. After the modulators, only the real part is transferred. The adaptive filters (AF) are real filters. They are adjusted using the error at each adaptive filter output. In the demodulator and the synthesis filter bank, the signals are represented with complex number. After the synthesis filter bank, only the real part is transferred.

Figure 2 shows examples of the signal spectra at the analysis filter bank output and the modulator output. The number of sub-bands M is 6. The frequency of 1 means the sampling rate f_s. The output of $F_i(z)$, $i = 0 \sim 5$, are shown in Fig.2(a), and one of them, that is the $F_1(z)$ output, is shown in Fig.2(b). This spectrum is sifted to the origin by the modulator as shown in Fig.2(c). By taking the real part of the modulator output, the symmetry part appears as shown in Fig.2(d). Finally, this real signal is down sampled by f_s/K, $K = 4$, as shown in Fig.2(e). The spectrum is expanded over a whole band $0 \le f \le f_s/K$. However, aliasing does not occur. K is not the same as M. The signal spectra in the demodulator and the synthesis filter bank are the same, in the reversed order, and explanation is omitted here.

3. Polyphase and FFT Realization

Modulation Sub-band Adaptive Filter

The analysis filter bank is taken into account for mathematical explanation here. The same derivation is possible for the synthesis filter bank. The fundamental low-pass filter (LPF) is denoted

$$F(z), \qquad z = exp(j2\pi f/f_s) \tag{1}$$

f_s is the sampling rate. $F_i(z)$ are obtained by sifting $F(z)$ as follows:

$$F_i(z) = F(z_i), \qquad i = 0 \sim M-1 \tag{2}$$

$$z_i = exp(\frac{j2\pi(f - f_s/4M - if_s/2M}{f_s}) \tag{3}$$

If the transient bandwidth is given by $2\Delta f$, then a single sub-band occupies $f_s/2M + 2\Delta f$. Thus, the minimum sampling rate after down sampling is

$$f_{sl} = 2(f_s/2M + 2\Delta f) \tag{4}$$

Furthermore, the following relations are held.

$$f_{sl} = f_s/K \tag{5}$$

$$\Delta f = \frac{f_s}{4}(\frac{1}{K} - \frac{1}{M}) \tag{6}$$

Next, the carrier signal applied to the modulator is given by

$$c_i(n) = exp[\frac{j2\pi(-\Delta f + if_s/2M)n}{f_s}] \tag{7}$$

Polyphase & FFT Realization

Equation (3) is modified as follows:

$$z_i = exp[\frac{j2\pi(f - f_s/4M - if_s/M)}{f_s}] \tag{8}$$

The shifting by $if_s/2M$ is replaced by if_s/M, which can provide the same spectram arrangement as shown in Fig.2 [6]. The relation between i and the sub-band number is also changed. In this paper, FIR filters are used for $F(z)$. However, the following discussion can be applied to IIR filters in the same way [8].

First, F(z) is expanded as follows:

$$F(z) = F_{l,0}(z^K) + z^{-1}F_{l,1}(z^K) + \dots + z^{-(K-1)}F_{l,K-1}(z^K) \quad (9)$$

The frequency shifting in Eq.(8) becomes

$$\begin{aligned} z_i^K &= exp[\frac{j2\pi K(f - f_s/4M - if_s/M)}{f_s}] \\ &= exp(\frac{j2\pi Kf}{f_s})exp(\frac{-j2\pi K}{4M})exp(\frac{-j2\pi iK}{M}) \\ &= z^K w^{-1/4} w^{-i}, \\ & w = exp(j2\pi K/M) \end{aligned} \quad (10)$$

$$z_i^{-j} = z^{-j} v^{j/4} v^{ij}, \; v = exp(j2\pi/M), \quad j = 0 \sim K-1 \quad (11)$$

Substuting Eqs.(10) and (11) into Eqs.(2) and (9), $F_i(z)$ is expressed as

$$\begin{aligned} F_i(z) &= F(z_i) = \sum_{j=0}^{K-1} z_i^{-j} F_{l,j}(z_i^K) \\ &= \sum_{j=0}^{K-1} z^{-j} v^{j/4} v^{ij} F_{l,j}(z^K w^{-1/4} w^{-i}) \end{aligned} \quad (12)$$

$$F_{i,l,j}(z^K) = F_{l,j}(z^K w^{-1/4} w^{-i}) \quad (13)$$

Figure 3 shows a block diagram to implement Eq.(12). The term $z^{-j}v^{j/4}$ can be commonly used by all $F_i(z)$, $i = 0 \sim M-1$. However, the other term $v^{ij}F_{l,j}(z^{-K}w^{1/4}w^i)$ are different for each sub-band. How to simplify this part is the next problem.

Polyphase Filter Realization

In Eq.(12), $w^{1/4}$ is common for all $F_{i,l,j}(z^K)$, then it can be combined with their coefficients. The other term $w^i = exp(j2\pi iK/M)$ is different from the conventional filter bank [6]. If $K = M$, then this term is unity, and becomes the same for all $F_i(z)$. This means that $F_{i,l,j}(z^k)$ are common for all $F_i(z)$. In this case, v^{ij} is realized as the discrete Fourier transform (DFT), and if M equals to the power of 2, DFT can be implemented by FFT [6].

However, when the number of sub-bands M is not equal to the sampling rate reduction K, w^i remains. How to implement this term with less computation is described in the following.

The sub-filters $F_{i,l,j}(z^{-K})$ is expressed as

$$\begin{aligned} F_{i,l,j}(z^K) &= h_{l,j,0} + h_{l,j,1}w^i z^{-K} + h_{l,j,2}w^{2i}z^{-2K} \\ &+ \dots + h_{l,j,P-1}w^{(P-1)i}z^{-(P-1)K} \\ &= \sum_{p=0}^{P-1} h_{l,j,p}w^{pi}z^{-pK} \end{aligned} \quad (14)$$

where, $h_{l,j,p}$ already includes $w^{1/4}$. Figure 4 shows a block diagram of $F_{i,l,j}(z^K)$. w^{pi} are separated from $h_{l,j,p}$. Letting the output of $h_{l,j,p}z^{-pK}$ be $u(j,p)$, the output of $F_i(z)$ becomes

$$\sum_{p=0}^{P-1} w^{pi}u(p,i) \quad (15)$$

This term can be shared by all $F_i(z)$, $i = 0 \sim M-1$. Thus, this term can be implemented as DFT.

From this figure and Eq.(14), if P is equal to M, then the term $w^{pi} = exp(j2\pi piK/M)$ is equivalent to DFT, of course the kernel, that is $w = exp(j2\pi K/M)$ is different from the original DFT, whose kernel is $w = exp(j2\pi/M)$. However, if M is the power of 2, FFT can be applied to the former in the same way.

Combining the block diagrams shown in Figs.3 and 4, a whole block diagram of the analysis filter bank, whose channel filters are expressed by Eq.(16), is composed as shown in Fig.5.

$$F_i(z) = \sum_{j=0}^{K-1} z^{-j}v^{j/4}v^{ij} \sum_{p=0}^{P-1} h_{l,j,p}w^{pi}z^{-pK} \quad (16)$$

Another point is the condition $P = M$ required above. This condition can be relaxed into $P = nM$, n is integer. For example, when $P = 2M$, Eq.(15) can be rewritten as follows:

$$\sum_{p=0}^{P-1} w^{pi}u(p,i) = \sum_{p=0}^{M-1} w^{pi}[u(p,i) + u(p+M,i)] \quad (17)$$

Because

$$w^{(p+M)i} = w^{pi}, \quad w = exp(j2\pi K/M) \quad (18)$$

Let the filter length of $F(z)$ be N. Since $P = N/K$, the condition $P = nM$ can be replaced by

$$N = nMK \qquad (19)$$

For example, $M = 8$, $K = 6$, then $N = 48n$, that is $N = 48, 96, 144$. N is required to be selected around these numbers.

Computational Complexity

Computational complexity of the analysis filter bank is evaluated here. In Fig.5, $v^{j/4}$ requires $2K$ real multiplications at the sampling rate f_s. $F_{l,j}(Z^K)$ requires $4N$ real multiplications at f_s/K, FFT requires $2KMlog_2M$ real multiplications at f_s/K. Finally, v^{ij} needs $4KM$ real multiplications at f_s/K. Totally, the analysis filter bank needs

$$\begin{aligned} Q_1 &= 2K + 4N/K + 2Mlog_2M + 4M \\ &\quad \textit{real multiplications at } f_s \qquad (20) \end{aligned}$$

On the other hand, the conventional polyphase & FFT method, in which $K = M$, requires

$$Q_2 = 2M + 4N/M + 2log_2M \qquad (21)$$

Furthermore, the direct realization at f_s requires

$$Q_3 = NM \quad \textit{real multiplizations at } f_s \qquad (22)$$

For example,
(1) when $N = 50$, $M = 8$, $K = 6$, then $Q_1 = 115$, $Q_2 = 47$ and $Q_3 = 400$.
(2) when $N = 100$, $M = 8$, $K = 6$, then $Q_1 = 159$, $Q_2 = 72$ and $Q_3 = 800$.
The proposed method requires more computations than the conventional polyphase & FFT method, which cannot be used in sub-band adaptive filters due to aliasing, however, it can save computations drastically compared with the direct realization.

5. Conclusions

A sub-band adaptive filter, in which modulation and demodulation are employed, can minimize the sampling rate, while no aliasing occurs. In this paper, a polyphase and FFT realization, which is different from that for the transmultiplexer, is proposed. In this case, the polyphase filters are different for the sub-bands due to mismatch between M and K. In the proposed method, the polyphase filters are divided into a transversal filter and an FFT. They can be shared by all the sub-bands. Computational complexity can be reduced.

Reference

[1] J.Cezanne and W.Kellerman,"Elliptic filters for the design of digital filter banks with nearly constant magnitude", Signal Processing, vol.14, pp.11-24, 1988.

[2] K.Nakayama and M.Tonomura,"A sub-band adaptive filter using oversampling filter banks (in Japanese)", Proc. IEICE, Tech., Meeting, vol.EA89-2, pp.7-14, Apr. 1989.

[3] M.G.Bellanger and J.L.Daguet,"TDM-FDM transmultiplexer: Digital polyphase and FFT", IEEE Trans. Commun., vol.COM-22, pp.1199-1205, Sept. 1974.

[4] A.Gilloire,"Experiments with sub-band acoustic cancelling for teleconferencing", Proc. IEEE ICASSP'87, pp.2141-2144, 1987.

[5] A.Gilloire and M.Vetterli,"Adaptive filtering in sub-bands", Proc. IEEE ICASSP'88, pp.1572-1575, 1988.

[6] P.P.Vaidyanathan, Multirate Systems and Filter Banks, Prentice Hall Inc., 1993.

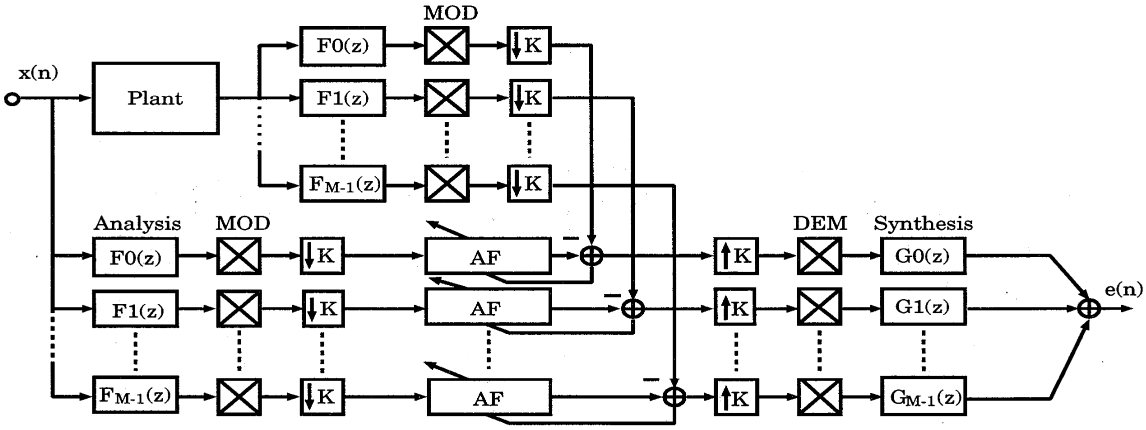

Fig.1 Modulation sub-band adaptive filter.

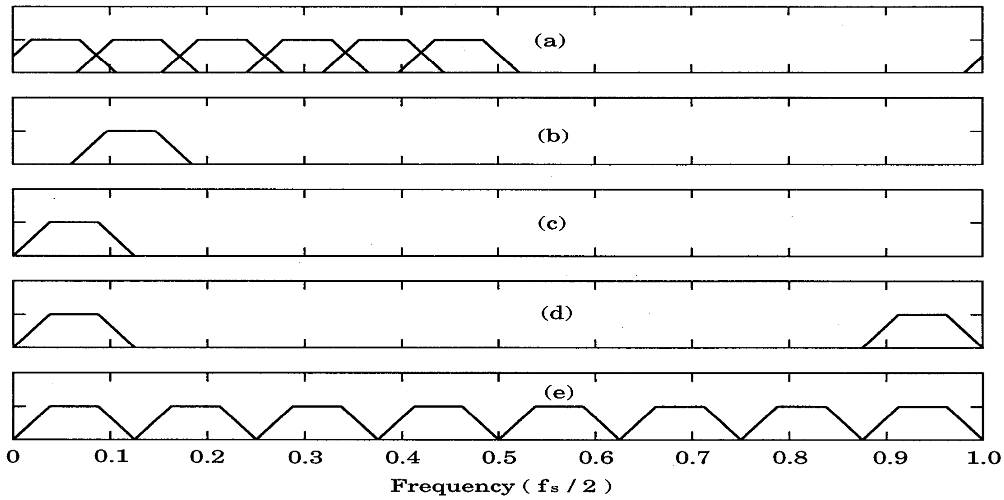

Fig.2 Signal spectra at analysis filter bank and modulator outputs.

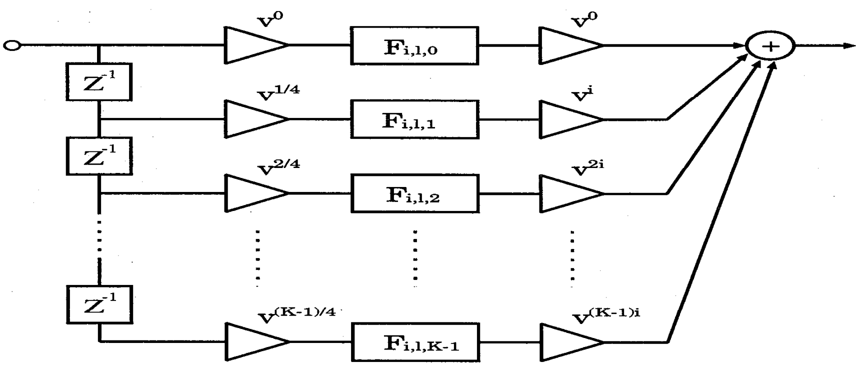

Fig.3 Block diagram of $F_i(z)$ expressed by Eq.(12).

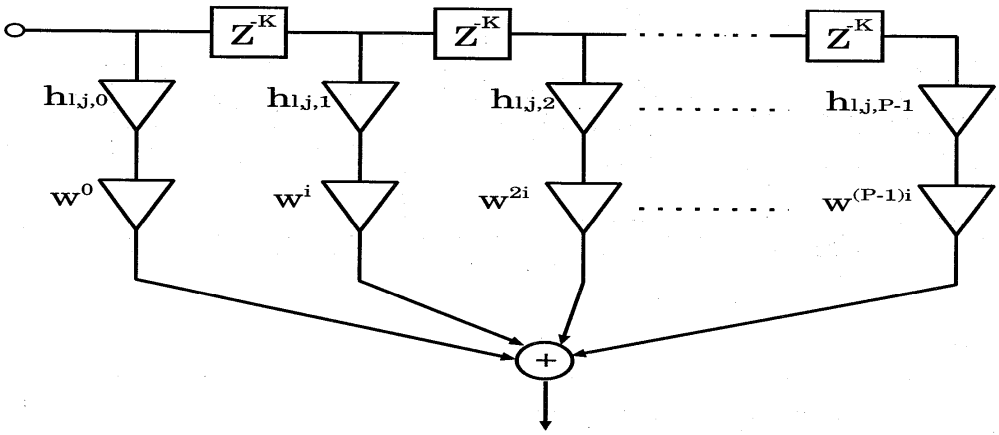

Fig.4 Block diagram of $F_{i,l,j}(z^K)$ expressed by Eq.(14).

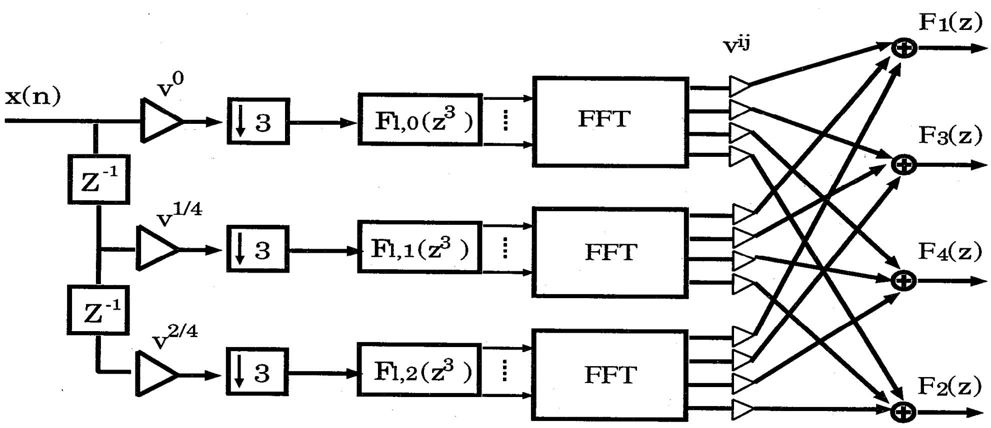

Fig.5 Proposed polyphase & FFT realization of analysis filter bank.