# IABP: Interval Arithmetic Backpropagation

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## Abstract

We present in this paper a new generalization of the BP (Backpropagation) algorithm by using interval arithmetic, this new algorithm permits the use of training samples and targets which can be indistinctly points and intervals. An interval vector in the input is translated into an interval vector in the output. It was already proposed an extension of BP to interval arithmetic [1], but it has the severe limitation that it can be only used for two class classification problems. This new algorithm can use any number of classification classes and represents a generalization of BP because every equation reduces to the normal BP equations in the case of point input vectors. This is due to a new definition of the error function:

$$E_P = \frac{1}{4} \cdot \sum_{K=1}^{Noutputs} \{ (t_{P,k}^L - O_{P,k}^L)^2 + (t_{P,k}^U - O_{P,k}^U)^2 \}$$

where $t$ is the target, $O$ the output, $P$ denotes a pattern, $L$ the lower limit of the interval and $U$ the upper limit.

The algorithm can be used to integrate expert's knowledge and training samples. The kind of expert's knowledge which can be integrated is "if ... then" rules of the type: if $X_{P,1} \subset [A_1, B_1]$ ... and $X_{P,n} \subset [A_n, B_n]$ then $X_P \in G_K$ , where $X_P$ is a pattern vector and $G_K$ its classification class, the intervals $[A_i, B_i]$ can be easily codified. Several two-dimensional simulation examples were presented.

It can be also used to efficiently represent "don't care attributes" [2]. We can codify a "don't care attribute" D by using an interval $[d_{min}, d_{max}]$, where $d_{min}$ represents the minimum of all the possible values of the attribute and $d_{max}$ the maximum. This codification overcomes an exponential increase in the training set which is needed in the case of normal BP and can be applied to discrete and continuous attributes. The same examples presented in [2] were investigated.

In general, the algorithm will add flexibility to the codification of inputs and targets. For example, in the case we have a strong subjectivity and imprecision (e.g., the codification of symptoms in a medical diagnosis problem) the use of intervals in the codification may reduce this subjectivity and imprecision, an imprecise or subjective input can be codified with a wider interval instead of a point.

## References

[1]    H. Ishibuchi and H. Tanaka. "An extension of the BP-Algorithm to Interval Input Vectors - Learning from Numerical Data and Expert's Knowledge-". IJCNN-91. Singapoure. pp.1588-1593.

[2]    H.M. Lee and C. Hsu. "The handling of Don't Care Attributes". IJCNN-91. Singapoure. pp. 1085-1091.