An Adaptive Nonlinear Function Controlled by Kurtosis for Blind Source Separation

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Abstract

In blind source separation, convergence and separation performances are highly dependent on a relation between probability density functions (pdf) of signal sources and nonlinear functions used in updating coefficients of a separation block. This relation was analyzed based on kurtosis κ_4*. It was suggested that* $\tanh y$ *and* y^3*, where* y *is the output, are useful nonlinear functions for super-Gaussian (*$\kappa_4 > 0$*) and sub-Gaussian (*$\kappa_4 < 0$*), respectively. In this paper, an adaptive nonlinear function is proposed. It has a form of* $f(y) = a \tanh y + (1-a)y^3/4$*, where* a *is controlled by the kurtosis of the output signal* $y_k(n)$*. It is assumed that the pdf* $p(y)$ *of the output signal satisfies the stability condition* $f(y) = -(dp(y)/dy)/p(y)$*. Based on this assumption, the parameter* a *and the kurtosis is related. This relation approximated by a function* $a = q(\kappa_4)$*. In a learning process,* $\kappa_4(n)$ *of the output signal is calculated at each sample* n*, and* $a(n)$ *is determined by* $a(n) = q(\kappa_4(n))$*. Then, the nonlinear function* $f(y)$ *is adjusted. Blind separation of music signals of 2–5 channels were simulated. The proposed method is superior to a method, which switches* $\tanh y$ *and* y^3 *based on polarity of* $\kappa_4(n)$*.*

1 Introduction

Recently, many kinds of information are transmitted and processed. At the same time, high quality is required. For this reason, signal processing including noise cancelation, echo cancelation, equalization of transmission lines, restoration of signals have been becoming very important technology. In some cases, we do not have enough information about signals and interference. Furthermore, their mixing and transmission processes are not well known in advance. Under these situations, blind source separation methods using statistical property of the signal sources have become important [1]-[5].

Jutten et all proposed a blind source separation algorithm based on statistical independence and symmetrical distribution of the signal sources [6]-[8]. Two stabilization methods have been proposed for Jutten's method [10],[17].

Convergence and separation performance are highly dependent on relation between a probability density function (pdf) of signal sources and nonlinear functions used in updating parameters in a separation block. Optimum nonlinearity has been discussed based on kurtosis. If the separation of signals of a certain class of distributions is the goal, the literature suggests to apply nonlinearities of the form $f(y) = ay^3$ for sub-Gaussian signals and $f(y) = a\tanh(y)$ for super-Gaussian signals, where y is the output and a is scalar used to adjust the output power. Another function is fixed to $g(y) = y$ [14],[15],[16].

In this paper, we propose an adaptive nonlinear function controlled by kurtosis. The pdf of the outputs are assumed to satisfy $f(y) = -(dp(y)/dy)/p(y)$ for the given nonlinear function $f(y)$. Kurtosis κ_4 is calculated using $p(y)$. Furthermore, $f(y)$ is controlled by κ_4. Multi-channel blind separation using music signals were simulated, and usefulness of the proposed method will be evaluated.

2 Network and Learning Algorithm

2.1 Network Structure

In this paper, the fully recurrent network shown in Fig.1 is taken into account. The number of the signal sources, the sensors and the outputs are all the same. The signal sources $s_i(n), i = 1, 2, ..., N$ are linearly combined using unknown weights a_{ji}, and are sensed at N points, resulting in $x_j(n)$. A general form is

$$x_j(n) = \sum_{i=1}^{N} a_{ji} s_i(n) \tag{1}$$

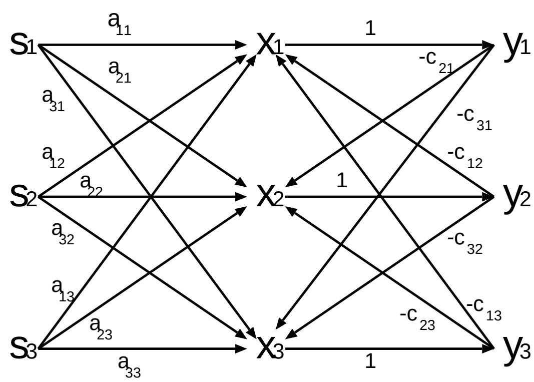

Figure 1: Block diagram for 3 signal sources, 3 sensors and 3 separations (3-3-3 model).

The output of the separation block $y_j(n)$ is given by

$$y_j(n) = x_j(n) - \sum_{\substack{k=1 \\ \neq j}}^{N} c_{jk} y_k(n) \quad (2)$$

This relation is expressed using vectors and matrices in the case of $N = 3$ as follows:

$$\boldsymbol{x}(n) = \boldsymbol{A}\boldsymbol{s}(n) \quad (3)$$

$$\boldsymbol{y}(n) = \boldsymbol{x}(n) - \boldsymbol{C}\boldsymbol{y}(n) \quad (4)$$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (5)$$

$$\boldsymbol{C} = \begin{bmatrix} 0 & c_{12} & c_{13} \\ c_{21} & 0 & c_{23} \\ c_{31} & c_{32} & 0 \end{bmatrix} \quad (6)$$

$\boldsymbol{A}$ is an unknown mixing matrix. From these expressions, a relation between the signal sources and the separation outputs becomes

$$\boldsymbol{y}(n) = (\boldsymbol{I} + \boldsymbol{C})^{-1}\boldsymbol{x}(n) = (\boldsymbol{I} + \boldsymbol{C})^{-1}\boldsymbol{A}\boldsymbol{s}(n) \quad (7)$$

The following matrix can be regarded as a separation matrix.

$$\boldsymbol{W} = (\boldsymbol{I} + \boldsymbol{C})^{-1} \quad (8)$$

In order to evaluate separation performance, the following matrix is defined.

$$\boldsymbol{P} = \boldsymbol{W}\boldsymbol{A} \quad (9)$$

If $\boldsymbol{P}$ has a nonzero element of each raw and column, and the number of nonzero elements is N, then the signal sources s_1, s_2 and s_3 are completely separated at the outputs y_1, y_2 and y_3. One example is shown here.

$$\boldsymbol{P} = \begin{bmatrix} 0 & p_{12} & 0 \\ p_{21} & 0 & 0 \\ 0 & 0 & p_{33} \end{bmatrix} \quad (10)$$

2.2 Learning Algorithm

In this paper, a learning algorithm, which uses all output signals to update c_{ij}, is employed. The update processes for feedforward and feedback types are given by [11],[12],[13],[14],

Feedforward Network

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + \eta(n)[\boldsymbol{\Lambda}(n) - f(\boldsymbol{y}(n))\boldsymbol{y}(n)^T]\boldsymbol{W}(n) \quad (11)$$

Fully Recurrent Network

$$\boldsymbol{C}(n+1) = \boldsymbol{C}(n) + \eta(n)[\boldsymbol{C}(n) + \boldsymbol{I}][\boldsymbol{\Lambda}(n) - f(\boldsymbol{y}(n))\boldsymbol{y}^T(n)] \quad (12)$$

$\boldsymbol{\Lambda}(n)$ is any positive-definite scaling diagonal matrix. In Eq.(12), for instance, $c_{ik}(n)$ is updated as,

$$c_{ik}(n+1) = c_{ik}(n) + \eta \sum_{j=1}^{N} c_{ij}(n) b_{jk}(n) \quad (13)$$

$$c_{ii}(n) = 0$$

$$b_{jk}(n) = -f_j(y_j(n)) y_k(n) \quad (14)$$

$$b_{jj}(n) = 1$$

Thus, $c_{ik}(n)$ is updated using overall output information, that is $y_k(n)$ and $y_j(n), j = 1, 2, \cdots, N, \neq k$. $b_{jk}(n) = f(y_j(n))y_k(n)$ is weighted with $c_{ij}(n)$. This weighting is the same for the elements in the ith row of $\boldsymbol{C}(n+1)$.

2.3 Optimum Nonlinear Functions

Optimum nonlinear functions are assigned based on kurtosis κ_4 as follows: [16]

$$\text{Kurtosis}: \quad \kappa_4 = \frac{E[(y-\bar{y})^4]}{E^2[(y-\bar{y})^2]} - 3 \quad (15)$$

$$\text{Sub-Gaussian}: \kappa_4 < 0 \quad f(y) = ay^3 \quad (16)$$

$$a = \frac{1}{\kappa_4 + 3} \quad (17)$$

$$\text{Super-Gaussian}: \kappa_4 > 0 \quad f(y) = a \tanh y \quad (18)$$

$$a = \frac{1}{E[y \tanh y]} \quad (19)$$

a is a scalar used to adjust the output power.

3 Adaptive Nonlinear Functions

3.1 Stability Condition

A relation between the pdf of the output signals and the nonlinear functions, which satisfies the local stability condition, is expressed as [16],

$$f(y) = -\frac{dp(y)/dy}{p(y)} \tag{20}$$

$p(y)$ is the pdf of the output signals.

3.2 Relation between Nonlinear Function and pdf of Output Signals

An adaptive nonlinear function proposed in this paper is given by

$$f(y) = a \tanh y + (1-a)\frac{y^3}{4} \tag{21}$$

The useful nonlinear functions $\tanh y$ and y^3 were derived for super-Gaussian and sub-Gaussian, respectively, based on the stability condition [16]. These functions are taken into our method. By synthesizing the nonlinear function $f(y)$ as a linear combination of these functions, it can cover a wide range of kurtosis, that is the pdf of the signal sources. The other nonlinear function is fixed to

$$g(y) = y \quad \text{fixed} \tag{22}$$

Assumption

The pdf of the output signal is assumed to satisfy the following local stability condition [13] for the nonlinear function $f(y)$ given by Eq.(21).

$$f(y) = -\frac{dp(y)/dy}{p(y)} \tag{23}$$

The above equation is solved by substituting $f(y)$ given by Eq.(21).

pdf of Output Signal

A solution of Eqs.(21) and (23) is obtained as follows:

$$p(y) = e^{-\left[a(\log\cosh y + 0.25) + (1-a)\left(\frac{y^4}{16}+0.45\right)\right]} \tag{24}$$

In the above equation, 0.25 and 0.45 are used to normalize $p(y)$.

Kurtosis

Kurtosis κ_4 defined by Eq.(15) is calculated using $p(y)$ obtained in the above.

$$\begin{aligned}\kappa_4 &= \frac{E[(y-\bar{y})^4]}{E^2[(y-\bar{y})^2]} - 3 \\ &= \frac{\int y^4 p(y)dy}{\left(\int y^2 p(y) dy\right)^2} - 3 \end{aligned} \tag{25}$$

Relation between Kurtosis and a

First, κ_4 is numerically calculated for given a by Eq.(25). a is changed from 0 to 1. This numerical relation is approximated by

$$a = \frac{1}{1 - e^{-4\kappa_4 - 1.2}} \tag{26}$$

-4 and -1.2 in the above are used to approximate the numerical relation. Furthermore, the above equation is expressed at each sample n as follows:

$$a(n) = q(\kappa_4(n)) \tag{27}$$

3.3 Nonlinear Function Control

Kurtosis of Output Signals

In a learning process, Eq.(25) is calculated by the following iterative integration.

$$\begin{aligned}
\bar{y}(n) &= (1-\alpha)\bar{y}(n-1) + \alpha y(n) &(28)\\
\kappa(n) &= (1-\alpha)\kappa(n-1) + \alpha(y(n)-\bar{y}(n))^4 &(29)\\
\sigma^2(n) &= (1-\alpha)\sigma^2(n-1) + \alpha(y(n)-\bar{y}(n))^2 &(30)\\
\kappa_4(n) &= \frac{\kappa(n)}{\sigma^4(n)} - 3 &(31)\\
0 &< \alpha \ll 1 &(32)
\end{aligned}$$

Linear Weight $a(n)$

$a(n)$ is calculated using $\kappa_4(n)$ obtained above by

$$a(n) = q(\kappa_4(n))$$

Nonlinear Function Control

Finally, the nonlinear function is controlled as

$$\begin{aligned}
f(y(n)) &= a(n)\tanh y + (1-a(n))\frac{y^3(n)}{4} &(33)\\
g(y(n)) &= y(n) \quad \text{fixed} &(34)
\end{aligned}$$

3.4 Stability Analysis

In this method, the local stability condition is directly used to relate the pdf of the output signal and the nonlinear function. Although the nonlinear functions are properly selected, the pdf obtained by Eq.(23) is not exactly the same as the actual distribution of the output signal. This mismatch will cause some instability.

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4 Numerical Analysis

Adaptive Nonlinear Function

Figure 2 shows the nonlinear function given by Eq.(21), where a is changed from 0 to 1.

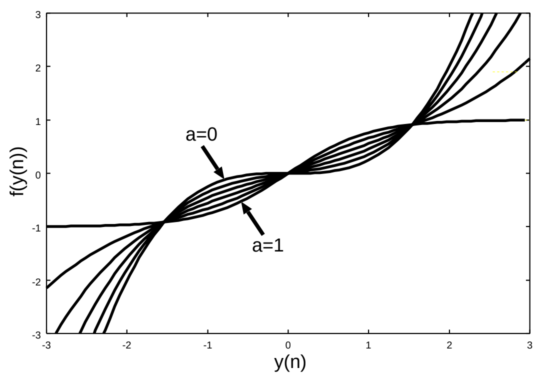

Figure 2: Adaptive nonlinear function given by Eq.(21).

pdf of Output Signal

Figure 3 shows the pdf of the output signal calculated by Eq.(24). Small a provides sub-Gaussian and large a gives super-Gaussian.

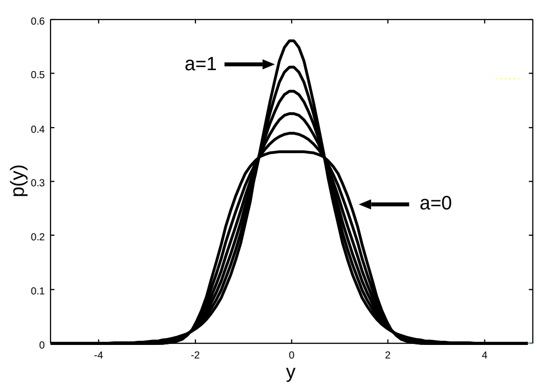

Figure 3: pdf of output signal calculated by Eq.(24). a is changed from 0 to 1.

Relation between Kurtosis and a

Figure 4 shows relations between the kurtosis and a. "Numerical relation" is calculated by Eq.(25) using numerical data of a. "Approximate" indicates the function given by Eq.(26), which approximates the numerical relation. After approximating the function $a = q(\kappa_4)$, $a(n)$ can be obtained using $\kappa_4(n)$, which is calculated using the output signal distribution as shown in Eqs.(28)–(31), at each sample n.

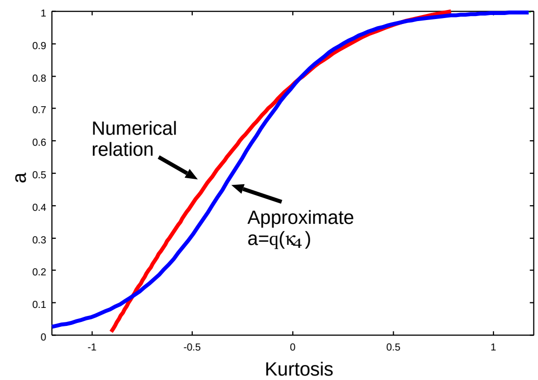

Figure 4: Relation between kurtosis and a.

Relation between pdf and Actual Distribution

Figure 5 shows the kurtosis κ_4 calculated following Eqs.(28)–(31) using the output signal $y(n)$ in the actual learning process. Signal sources are music signals. α is set to 0.0005, that is, approximately 2,500 samples are integrated to calculate κ_4.

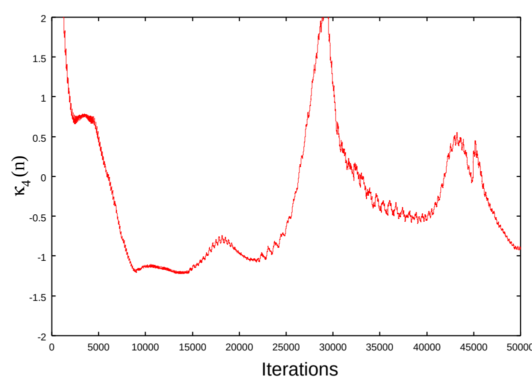

Figure 5: Kurtosis calculated by using output signal in learning process.

Figure 6 shows the pdf calculated by Eq.(24) and actual distribution of the output signal. Average values of κ_4 in a 2,500 sample interval shown in Fig.5 are used in Eq.(26). Furthermore, a is used in Eq.(24) to calculate the pdf of the output signal.

Roughly speaking, they are similar to each other. However, still they are not exactly the same. This relation is highly dependent on the nonlinear function given by Eq.(21). If we select another type of functions, then the relation will be changed. This point should be more investigated.

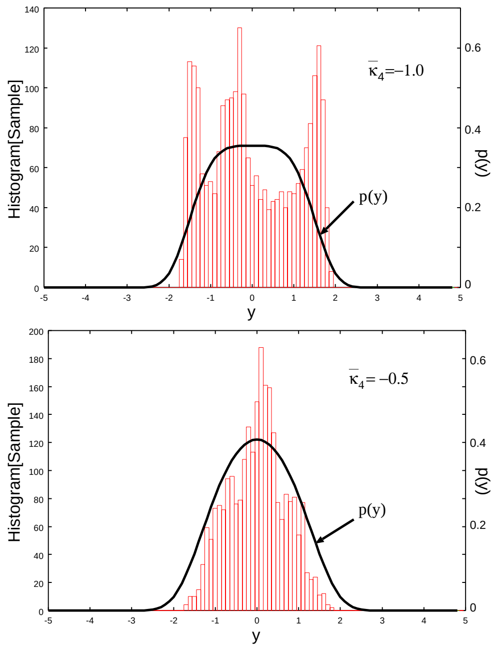

Figure 6: Relation between pdf and actual distribution of output signal for different kurtosis. Kurtosis is averaged over 2,500 samples.

5 Simulation of Blind Separation

5.1 Simulation Conditions

Evaluation of Separation

Separation performance is evaluated by the following SNR.

$$\mathrm{SNR} = 10 \log \frac{\sum_{i,j \in \Omega_1} p_{ij}^2}{\sum_{i,j \in \Omega_2} p_{ij}^2} \tag{35}$$

p_{ij} are elements of $\boldsymbol{P}$ in Eq.(9). Ω_1 includes the elements of the separated signal sources, and Ω_2 includes the elements of the cross terms.

Mixing Matrix

$$\boldsymbol{A}_{3ch} = \begin{bmatrix} 1.0 & 0.6 & 0.5 \\ 0.3 & 1.0 & 0.7 \\ 0.4 & 0.5 & 1.0 \end{bmatrix} \tag{36}$$

$$\boldsymbol{A}_{5ch} = \begin{bmatrix} 1.0 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 1.0 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1.0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 1.0 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 1.0 \end{bmatrix} \tag{37}$$

Signal Sources

Music signals are mainly used, because their pdf dynamically change, and their kurtosis $\kappa_4(n)$ take positive and negative values. Therefore, adjusting the nonlinear functions becomes very important.

5.2 Separation Performances

Figures 7, 8 show learning curves for 3ch and 5ch music signal source separation. Figure 9 shows the case of 3ch including two voices and one white noise. The vertical axis is SNR defined by Eq.(35). The switching method selects ether $\tanh y$ or y^3 for the nonlinear function based on $\kappa_4(n) > 0$ or $\kappa_4(n) < 0$, respectively. This method is regarded as a conventional approach [16].

In all cases, the proposed is superior to the conventional. Usefulness of the proposed is evident when the number of channels is increased. Furthermore, this method is still useful for voice signals, whose kurtosis is not so widely distributed.

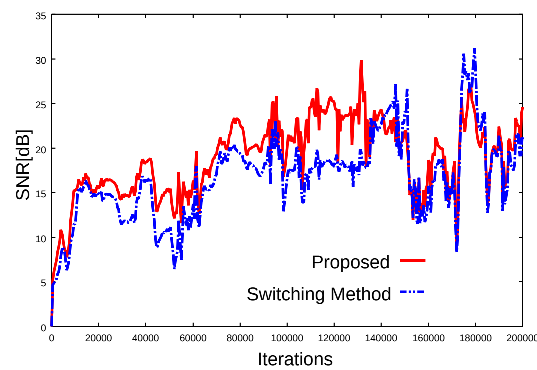

Figure 7: Learning curves for 3-channel music signal source separation.

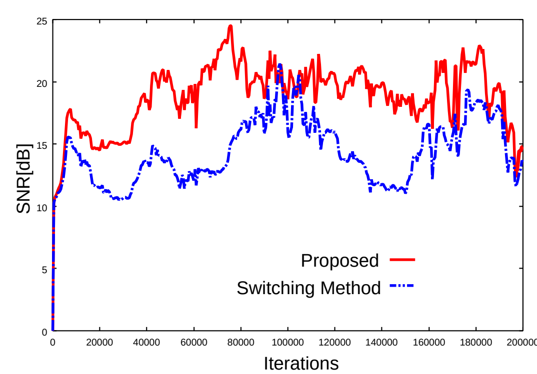

Figure 8: Learning curves for 5-channel music signal source separation.

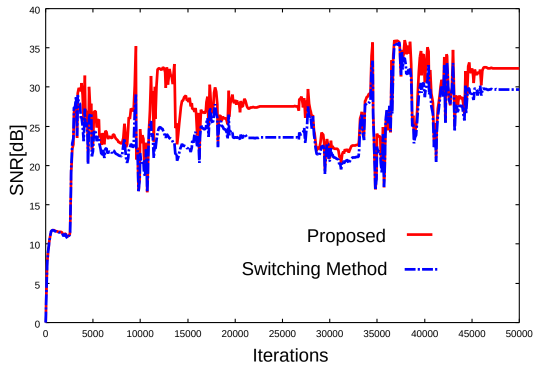

Figure 9: Learning curves for 3-channel signal source separation, including two voices and one white noise.

6 Conclutions

An adaptive nonlinear function has been proposed. It is formed as a linear combination of $\tanh y$ and y^3. The linear weight is controlled by the kurtosis. The pdf of the output signal is related to the nonlinear function based on the stability condition. Simulation of blind separation of 3ch and 5ch music signals have been demonstrated. The proposed is superior to another method, which switches the nonlinear function based on polarity of the kurtosis.

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