A Tap Assignment Method using MSE and Tail Taps for Sub-band Adaptive Filters

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Abstract- A new tap assignment method for sub-band adaptive filters is proposed. At the beginning of adaptation, the tap assignment is controlled by the mean-squared error(MSE) of each band. After the residual errors of all the bands become the same, the tap assignment is controlled by both the MSE and the tail tap weights of each sub-band adaptive filter. The new method in contrast to the method which uses only the MSE, allows taps to be correctly assigned in the event that the sub-band adaptive filters have substantially different convergence rates. The efficiency of the proposed method has been confirmed through computer simulation. The performance of the proposed method in case of a colored input signal is investigated. It is confirmed that, in the colored input signal case, the total residual error can be minimized even though the unknown system is not exactly identified. Therefore, the proposed method is suitable for applications, such as, echo cancellation and noise cancellation.

I Introduction

Sub-band adaptive filtering is an efficient technique for reducing computational requirements and for improving the convergence rate. In the conventional method, the numbers of taps of all the sub-band adaptive filters are set to be the same [1]-[5]. However, a uniform length for all the sub-band adaptive filters is not always the optimum. When the lengths of the impulse responses in all the bands are not the same, the number of taps will be insufficient in the bands which have longer impulse responses, or redundant in other bands which have shorter impulse responses.

In a recent study, we have proposed an Automatic-Tap-Assignment (ATA) sub-band adaptive filter configuration which enables automatic adjustment of the lengths of sub-band adaptive filters [6]. In the ATA sub-band adaptive filter, the numbers of taps of all the sub-band adaptive filters are controlled by the mean-squared errors (MSEs). The numbers of taps increase in the bands where large errors occur and decrease in the bands where small errors occur, until the MSEs in all the bands become the same. This tap assignment method is named the MSE method in this paper. It is confirmed that the ATA sub-band adaptive filter can obtain a smaller residual error compared to the conventional uniform-length sub-band adaptive filters.

The following problem remains in the MSE method. When the convergence rates in all the sub-band adaptive filters are substantially different, wrong tap assignment occurs during the adaptation process. Redundant taps may be assigned to the band for which the convergence rate is very slow.

This paper propose a new method to avoid assigning redundant taps to the band which has a very slow convergence rate. The efficiency of the proposed method is confirmed by computer simulation. The performance of the proposed method in the case of the colored input signal is also investigated.

II Configuration of ATA Sub-band Adaptive Filter

The configuration of the ATA sub-band adaptive filter is shown in Fig.1. $\mathbf{x}(n)$, $e_0(n)$ and $\mathbf{d}(n)$ denote a input signal, measurement error and desired response, respectively. $\mathbf{x}(n)$ and $\mathbf{d}(n)$ are divided into K bands by means of an analysis filter bank (AFB). $\mathbf{x}_1(n),\ldots, \mathbf{x}_K(n)$ and $\mathbf{d}_1(n),\ldots, \mathbf{d}_K(n)$ denote the components of $\mathbf{x}(n)$ and $\mathbf{d}(n)$ in the bands 1, ..., K. M denotes the decimation and interpolation rate. In order to avoid the influence of the aliasing components, over-sampling (K=2M) is used. This does not influence the efficiency of the ATA sub-band adaptive

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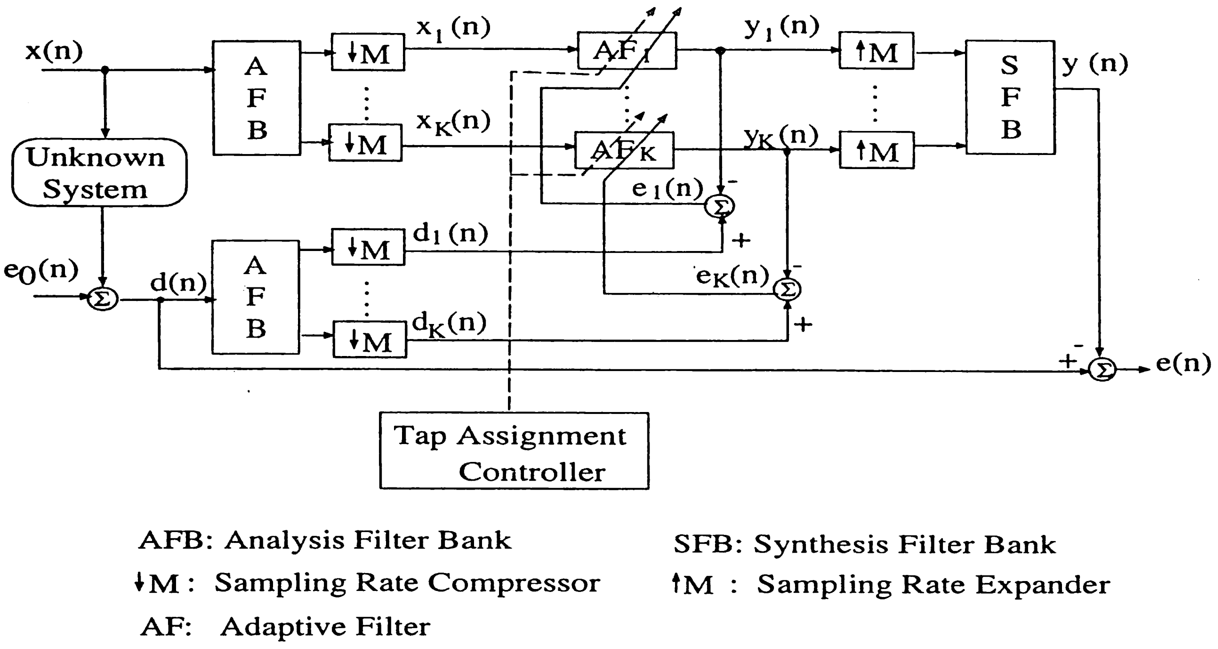

Figure 1: ATA sub-band adaptive filter configuration.

filter. The outputs of the sub-band adaptive filters $y_1(n)$, ..., $y_K(n)$ are synthesized using a synthesis filter bank (SFB) so as to obtain the total output $y(n)$. The difference between $d(n)$ and $y(n)$ is the total residual error $e(n)$. The tap weights of the adaptive filters in the bands 1, ..., K are updated to minimize the corresponding residual errors $e_1(n)$, ..., $e_K(n)$. The number of taps for each sub-band adaptive filter is not fixed. It is automatically adjusted by means of a tap assignment controller.

III A New Tap Assignment Method

In the MSE method, the tap assignment depends on the residual error in each band. In actual applications, the total number of taps is usually fixed due to hardware limitations. By increasing the number of taps in the band having the largest residual error, and decreasing the number of taps in the band having the smallest residual error, the residual errors in all the bands converge to the same value. As a result, the total residual error can be minimized. The optimal tap assignment is guaranteed only when all the bands have almost the same convergence rate. If the convergence rates are substantially different, then the band with slowest convergence rates may require redundant taps. The residual errors of the other bands are forced to be the same as the band which has the slowest convergence rate. If we can use a long adaptation time, then the optimal tap assignment can be obtained after all sub-band adaptive filters converge. However, the use of long adaptation time is not practical. Also, this kind of wrong tap assignment can not be detected by using only the MSE.

A new tap assignment method is proposed in order to solve this problem. At the beginning of adaptation, the MSE method is used. After all the bands have the same residual MSE, the tail tap weights are then taken into account in order to detect redundant taps. For simplifying description of the proposed method, a two-band case is used.

At the beginning of adaptation, the same number of taps is assigned to the two bands.

$$N_1(0) = N_2(0) = \frac{N}{2} \tag{1}$$

where N is the total number of taps. The MSE of the kth sub-band adaptive filter is calculated every M_0 samples.

$$E_k(n) = \frac{1}{M_0} \sum_{m=n-M_0+1}^{n} e_k^2(m), \qquad k = 1, 2, n = M_0, 2M_0, \cdots \tag{2}$$

If the MSE of band 1 is larger than that of band 2, then δ taps are subtracted from band 2 and added to band 1.

δ can be changed during the adjusting process. The initial values of δ taps are set to zero and can be expressed by the following formulas. If

$$E_1(n) > E_2(n)(1+\epsilon_1), \qquad (3)$$

then

$$\begin{aligned} N_1(n+1) &= N_1(n)+\delta \\ N_2(n+1) &= N_2(n)-\delta \end{aligned} \qquad (4)$$

$$\begin{aligned} \mathbf{w}_{1j}(n+1) &= \mathbf{w}_{1j}(n), \quad 0 \le j \le N_1(n) \\ \mathbf{w}_{1j}(n+1) &= 0, \quad N_1(n)+1 \le j \le N_1(n+1) \\ \mathbf{w}_{2j}(n+1) &= \mathbf{w}_{2j}(n), \quad 0 \le j \le N_2(n+1), \end{aligned} \qquad (5)$$

where ϵ_1 is a small positive constant which is introduced to avoid oscillation in the tap assignment. If the MSE of band 2 is larger than that of band 1, the tap assignment is similar to the above.

The tap assignment according to the MSEs continues until

$$|E_1(n) - E_2(n)| \le \epsilon_1, \qquad (6)$$

Then the mean-squared values of the N_T tail tap weights are calculated as follows:

$$w_{Tk}(n) = \frac{1}{N_T}\sum_{i=0}^{N_T-1} \mathbf{w}^2_{N_k(n)-i}, \; k = 1,2 \qquad (7)$$

where $N_k(n)$ is the number of taps of the kth band at the time of iteration n. The number of taps increases only in those bands which have large values both in $E_k(n)$ and $w_{Tk}(n)$, or alternatively when Eq.(3) and the following condition are satisfied,

$$w_{T1}(n) > w_{T2}(n)(1+\epsilon_2) \qquad (8)$$

The number of taps in band 1 increases according to Eq.(4). ϵ_2 has the same meaning as ϵ_1. The initial tap weights are shown in Eq.(5).

The number of taps of band 1 does not increase as long as the tail tap weights of band 1 are smaller than band 2, even though the residual error of band 1 become larger than that of band 2. Therefore, no redundant taps are assigned to bands having slow convergence rates.

IV Optimal Tap Assignment

Optimal tap assignment means the total residual error can be minimized by assigning the optimal taps to each sub-band adaptive filter, while keeping the total number of taps unchanged. If the residual error in each band is uncorrelated, then the total mean-squared residul error E(n) can be expressed as [7]

$$E(n) = \sum_{i=1}^{K} E_k(n) \qquad (9)$$

$E_k(n)$ is the mean-squared residul error of the kth-band as shown in Eq.(2) E(n) can be minimized by minimizing $E_k(n)$. $E_k(n)$ depends on $e_k(n)$, the residual error of the kth-band adaptive filter. We will discuss how $e_k(n)$ can be minimized. $e_k(n)$ is given by

$$\begin{aligned} e_k(n) &= \mathbf{d}_k(n) - \mathbf{w}_k^H(n)\mathbf{X}_k(n) \\ &= [\mathbf{w}_{k0}^H(n) - \mathbf{w}_k^H(n)]\mathbf{X}_k(n) + e_{0_k}(n) \\ &= \mathbf{e}_{w_k}(n)\mathbf{X}_k(n) + e_{0_k}(n) \end{aligned} \qquad (10)$$

where $[\cdot]^H$ denotes the Hermitian transpose, $\mathbf{w}_{k0}^H(n)$ denotes the ideal tap weights and $\mathbf{e}_{w_k}(n)$ denotes the tap-weight error vector of the kth-band adaptive filter. $\mathbf{X}_k(n)$ and $e_{0_k}(n)$ denote the input vector and measurement error components in the k-band. Eq.(10) shows that $e_k(n)$ depends on three components, $\mathbf{e}_{w_k}(n)$, $\mathbf{X}_k(n)$ and $e_{0_k}(n)$.

Tap-weight error vector $\mathbf{e}_{w_k}(n)$ dependents on the number of taps and tap-weights in the kth-band. If enough taps are assigned to the kth-band to cover the length of the impulse response in this band, and tap-weights are approximately equal to the value of the impulse response, then $\mathbf{e}_{w_k}(n)$ will become small. The adaptation of tap-weights is common in any adaptive filter. Our research emphasizes on how to assign the optimal number of taps to each band, when the total number of taps is given. As mentioned in section III, the number of taps in the kth-band is automatically adjusted according to the MSE and tail tap values so as to minimize $e_k(n)$.

When the input signal $\mathbf{x}(n)$ is colored, such as speech and music, $\mathbf{X}_k(n)$ varies depending on the spectrum of the input signal. $e_k(n)$ is large when $\mathbf{X}_k(n)$ is large, therefore, the number of taps in the kth-band increases in order to minimize $e_k(n)$. The total residual error can be minimized even though the unknown system is not exactly identified. Therefore, the ATA sub-band adaptive filter can be effectively applied to noise and echo cancellation.

It is customary to assume that the measurement error is white with zero mean and variance σ^2. The mean-squared value of $e_{0_k}(n)$ which is caused by the measurement error in each band is the same. The tap assignment in each band is biased by the same value. Thus the optimal tap ratios are different from that obtained when no measurement exists.

V Simulation Results and Discussions

A. Algorithm and Unknown System

Polyphase structures were used for the analysis and synthesis filter banks [8]. A 41-tap quadrature mirror filter (QMF) was used as a prototype filter for the two-band case. Reconstruction error of analysis/synthesis system is less than $\pm 0.11dB$. The normalized LMS algorithm [9] was used in adapting of the kth sub-band adaptive filter.

$$e_k(n) = d_k(n) - \mathbf{w}_k^H(n)\mathbf{X}_k(n), \qquad (11)$$

$$\mathbf{w}_k(n+1) = \mathbf{w}_k(n) + \frac{\alpha}{\varepsilon + |\mathbf{X}_k(n)|^2}\mathbf{X}_k(n)e_k^H(n) \qquad (12)$$

where $[\cdot]^H$ denotes the Hermitian transposition, $\alpha = 0.02$, and $\varepsilon = 10^{-10}$. The input signal x(n) is white noise with zero mean and variance $\sigma^2 = 0.082(-10.86dB)$. The amplitude of the transfer function and the impulse response of an unknown system are shown in Fig. 2.

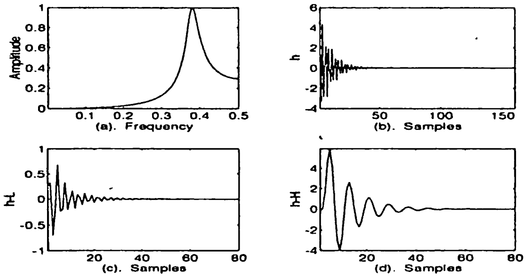

Figure 2: Amplitude and impulse response of the unknown system. (a) Amplitude-frequency response, (b) Impulse response in the full-band, (c) Impulse response in the low-band, (d) Impulse response in the high-band.

B. Relation Between Total Residual Error and Tap Assignment

The relation between the total residual error and the lengths of the sub-band adaptive filters is investigated. The simulation result is shown in Fig.3 . The abscissa denotes the number of taps in band 2. The ordinate denotes the total MSE, where the total MSE is defined to be the average MSE over the final 1000 samples of e(n). In the simulation, the number of total taps was set to be 50. The total MSE is $-13.74dB$ at $N_1(n) = 25$, for the uniform-length method. The total MSE decreases as $N_2(n)$ increases. At $N_2(n) = 42$, the total MSE attains the minimum value, $-21.91dB$. As $N_2(n)$ increases further, the total MSE also increases.

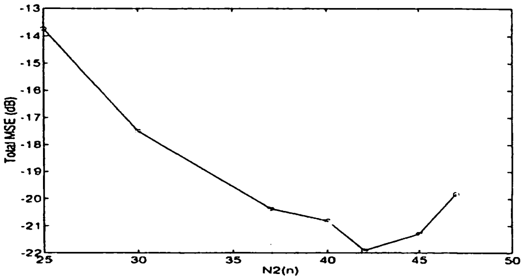

Figure 3: Relation between the total MSE and the number of taps.

C. Simulation Result With MSE Method

The simulation results of the ATA sub-band adaptive filter adjusted by the MSE method are shown in Fig.4. The convergence rate of band 1 is substantially slower than that of band 2. At the beginning of adaptation, the residual error of band 2 is larger than that of band 1, because of the longer impulse response in band 2. Therefore, the number of taps increases in band 2. After about 2000 iterations, the residual error of band 2 becomes smaller than that of band 1, so that the number of taps in band 2 decreases and the number of taps in band 1 increases. However, the large residual error of band 1 is caused by the slow convergence rate, and not by a shortage of taps. Thus, the MSE of band 1 can not be reduced by increasing the number of taps. As shown in Fig.4, the values in the tail of band 1 are almost zero. This means that increasing the number of taps in band 1 will not provide any improvement. The residual error of band 2 is forced to be the same as that of band 1 by decreasing the number of taps. The number of taps in band 1 and band 2 are 20 and 30 at 8000 iterations. This is far from the optimum value as shown in Fig.3.

D. Simulation Result With Proposed Method

The simulation results of the ATA sub-band adaptive filter adjusted by the proposed method is shown in Fig.5. ϵ_1 and ϵ_2 in Eq.(3) and (8) respectively

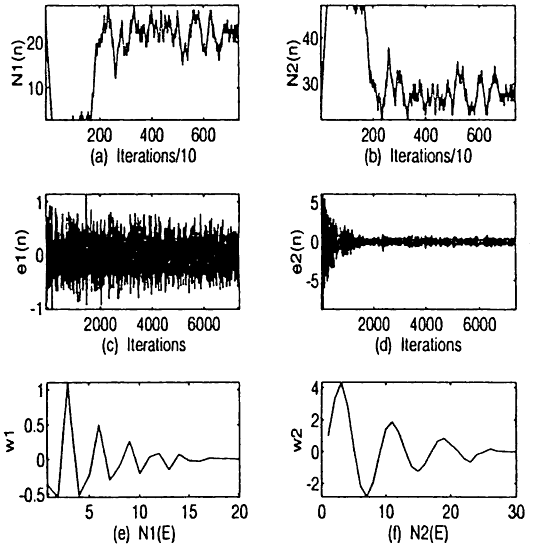

Figure 4: Simulation result using the MSE method. (a). Tap assignment in band 1, (b). Tap assignment in band 2, (c) Residual error of band 1, (d) Residual error of band 2, (e) Tap weights of band 1, (f) Tap weights of band 2.

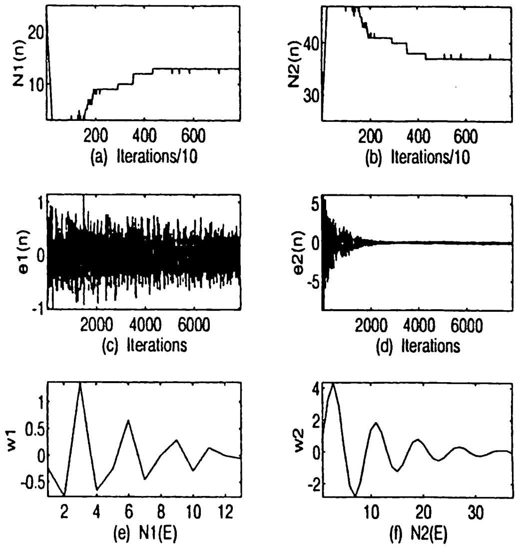

Figure 5: Simulation result using the proposed method.(a). Tap assignment in band 1, (b). Tap assignment in band 2, (c) Residual error of band 1, (d) Residual error of band 2, (e) Tap weights of band 1, (f) Tap weights of band 2.

are set to 0.01 in the simulation. At the beginning of adaptation, the tap assignment depends on the MSEs, and the performance is the same as shown in Fig.4. After about 2000 iterations, the residual errors of bands 1 and 2 become almost the same. Therefore, the tail tap criterion is then taken into account. The tap assignment is adjusted using both the MSEs and the tail tap values. As shown in Fig.5, there are no redundant taps in the tail of band 1. The residual error of band 2 is smaller than that of band 1. This difference in error is as a result of inherent difference in their convergence rates. A wrong tap assignment is avoided. The number of taps in band 1 and band 2 are 13 and 37 respectively after 8000 iterations. These values are near the optimum value as shown in Fig.3.

E. Performance for Colored Input Signal

In order to evaluate the total residual error and the system identification error of the ATA sub-band adaptive filter adapted using white and colored input signals, respectively, the process of Fig.6 is used[10]. For the first 8000 iterations, a white noise input signal is used so as to avoid a slow convergence rate. During the next 4000 iterations, the input signal is comprised of a white noise and a colored signal. The residual error is evaluated during this interval. The system identification error is evaluated after 8000 and 12000 iterations, where the system identification error is defined as the difference between the impulse response of the unknown system and the impulse response of the ATA sub-band adaptive filter.

A speech signal shown in Fig.7 is used as the colored input signal. The variance of the speech signal is the same as the white input signal, that is, $\sigma^2 = 0.082(-10.86dB)$. The signal power spectrum is concentrated in the low frequency band.

First the system identification error is evaluated. The system identification error is $-5.23dB$ after the first 8000 iterations. It becomes $-5.77dB$ after another 4000 iterations if a white noise input signal is used, and $-5.06dB$ if a speech input signal is used. These results shows that the preciseness of system identification is slightly degraded when the input signal is a speech signal.

After this, the total residual error for the speech input signal is evaluated for two cases; the ATA sub-band adaptive filter is adapted using the speech signal and white noise signal respectively. The mean-squared error after 4000 iterations is $-34.62dB$ if a speech input signal is used, and $-22.27dB$ if a white noise input signal is used. The results show that, for the colored input signal case, the total residual error can be minimized even though the system is not exactly identified. For applications such as noise cancellation and echo cancellation, the purpose is to minimize the total residual error. Therefore, the ATA sub-band adaptive filter is suitable for these applications.

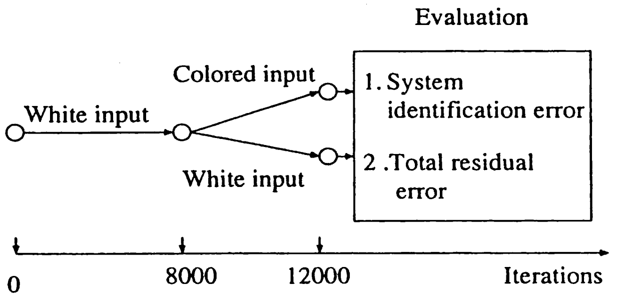

Figure 6: Evaluation of residual error and impulse response error for colored and white input signals.

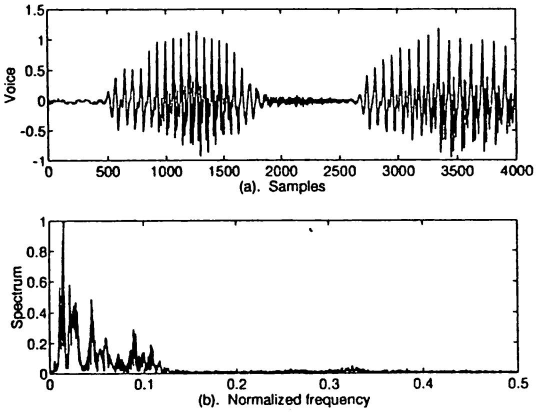

Figure 7: (a). Speech signal, (b). Spectrum of the speech signal.

VI Conclusion

A new tap assignment method has been proposed. At the beginning of adaptation, the tap assignment is controlled by the MSEs. After the residual errors of all the bands become the same, the tap assignment is controlled by both MSEs and tail tap weights of the sub-band adaptive filters. The number of taps increases only in the bands which have both large MSEs and tail tap weight values. The wrong tap assignment which occurs in the existing MSE method can be avoided in the proposed method.

In the colored input signal case, the total residual error can be minimized even though the unknown system is not exactly identified. Therefore, the proposed method is useful for applications, such as, echo cancellation and noise cancellation.

References

[1] W. Kellermann, "Analysis and design of multirate systems for of acoustical echos," in Proc. IEEE ISCAS'88, pp.2570-2573, Apr. 1988.

[2] A. Gilloire and M. Vetterli, "Adaptive filtering in subbands," in Proc. IEEE ICASSP'88, pp.1572-1575, Apr. 1988.

[3] H. Perez and F. Amano, "A new subband echo canceller structure," Trans. IEICE, vol. E73, no.10, pp.1625-1631, Oct.1990.

[4] J.Shynk, "Frequency-Domain and Multirate Adaptive Filtering," IEEE SP Magazine, pp.14-37, Jan. 1992.

[5] K. Nakayama and M. Tonomura, "A sub-band adaptive filter using oversampling filter banks," in Proc. of Tech. Meeting Electro Acoustics, IEICE, EA89-2, April 1989.

[6] Z. Ma, K. Nakayama and A.Sugiyama, "Automatic tap assignment in sub-band adaptive filter," IEICE Trans. Commun., vol. E76-B, no.7 pp.751-754, July 1993.

[7] H. Ochi, Y. Higa, S., Kinjo, " On the relationship of the cost function between full-band and sub-band adaptive filter," Proc. 1994 IEICE Spring Conference, A-202.

[8] R. E. Crochiere, and L. R. Rabiner, Multirate Digital Signal Processing, Prentice-Hall, Inc., Englewood Cliffs, N.J. 07632, 1983.

[9] B. Widrow and S. D. Stearms, Adaptive signal Processing, Prentice-Hall, Inc., Englewood Cliffs, N.J. 07632, 1985.

[10] Z. Ma, and K. Nakayama, " Performance of An Automatic Tap Assignment Adaptive Filter for Measurement Noise and Colored Input Signal," Proc. 7th Karuizawa Workshop on CAS, pp.751-754, April, 1994.